Preliminary Investigation of Acoustic Radiation from Large-Scale Turbulent Structures within a Two-Dimensional Shear Layer

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Outline

- Preliminary
- Introduction
- Mathematical Methods
- Preliminary Results
  - Computational Fluid Dynamics
  - Acoustic Radiation
- Future Research Direction

Preliminary

Brief Biography

University of Florida

- Asst. Prof. of Mech. & Aero. Eng.

Previously at **NASA**

- NASA Civil Servant from 2009 - 2016
- Research Aerospace Engineer

Education

- Ph.D. (NASA Grant)
  Aerospace Engineering, Penn State
- M.S. (NREL Grant)
  Aerospace Engineering, Penn State
- B.S., Mechanical Engineering, Michigan State University

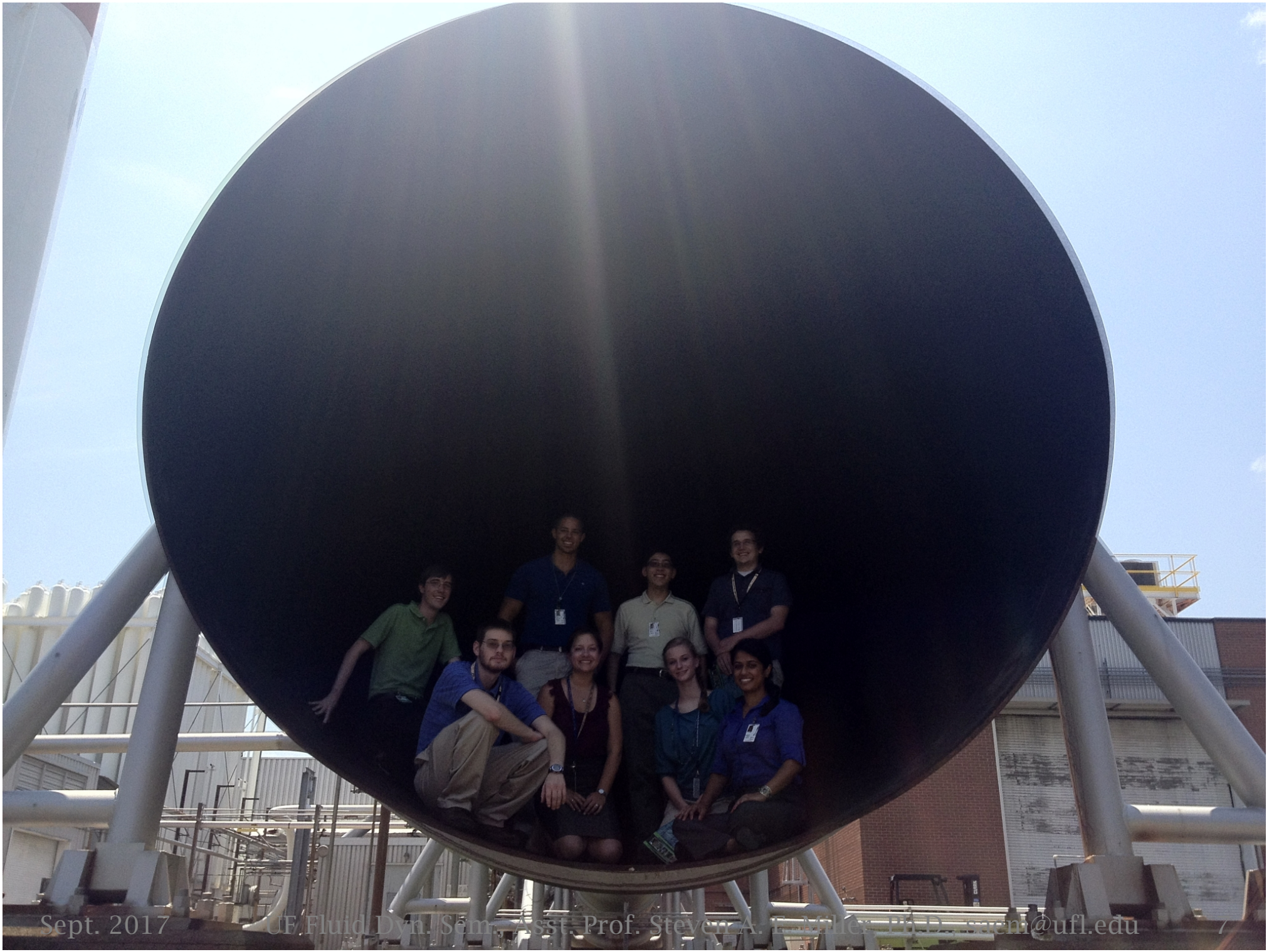
Research Overview

UF Herbert Wertheim College of Engineering

POWERING THE NEW ENGINEER TO TRANSFORM THE FUTURE

Steven A. E. Miller, Ph.D.
Assistant Professor of Mech. and Aero. Eng.
Ph.D., Penn State University, 2009

- Theoretical fluid dynamics, aeroacoustics, and turbulence
- Compressible turbulent jets, shock waves, shear layers, boundary layers, fluid structure interaction
- Nonlinear propagation of waves
- Applications – rocket and jet engines, external aerodynamics, wave propagation, sources of sound, and more…

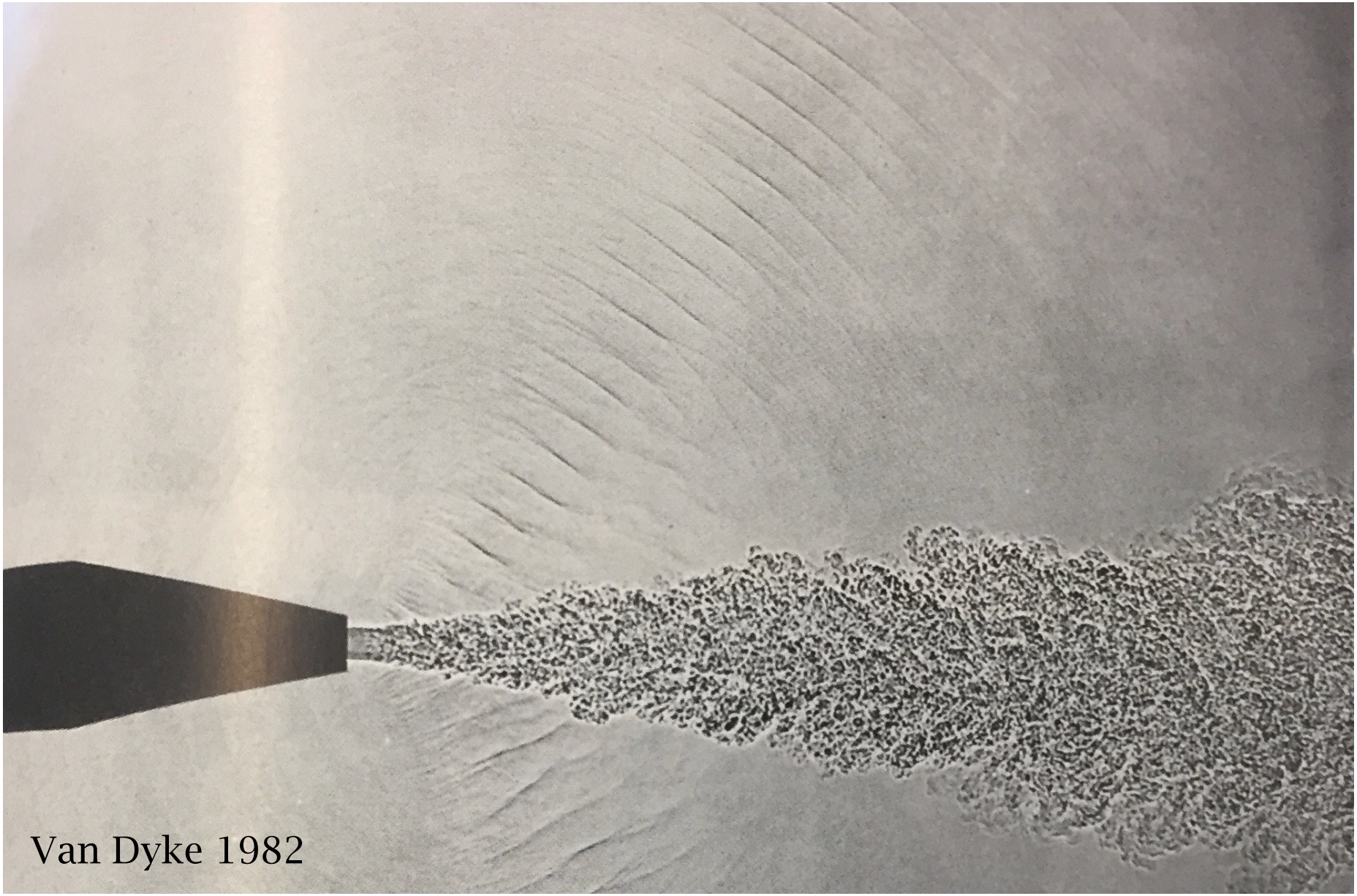


UF Student Research

- Carr, Alex – Jet structure interaction
- Endurthi, Vijay – Rotorcraft aeroacoustics
- Pager, Elisha - University Scholars, turbulent boundary layers, CFD class development
- Patel, Trushant – Aeroacoustic modeling
- Roberts, Kyle – Sonic boom and counter-flowing jets
- Shen, Weiqi – LES and decomposition of jets
- Wang, Wei - Multiphase flow of off-design supersonic jets (with Dr. Bala)

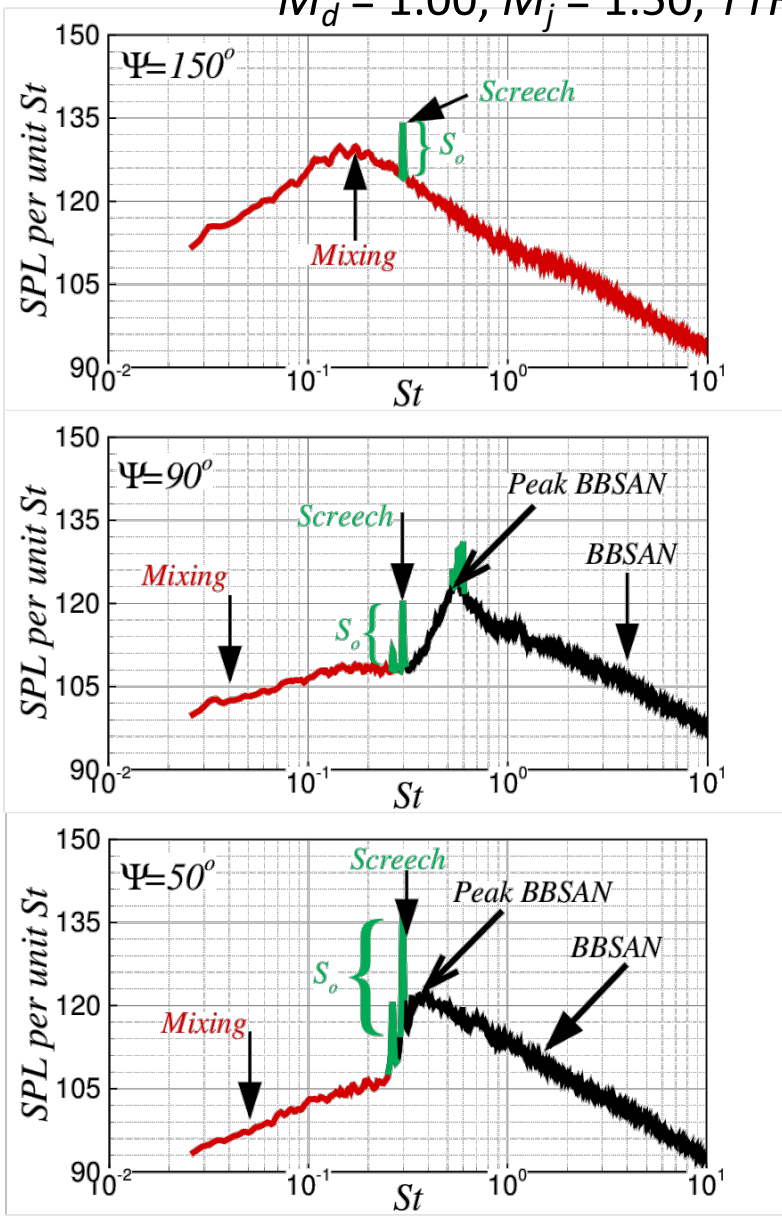
Introduction

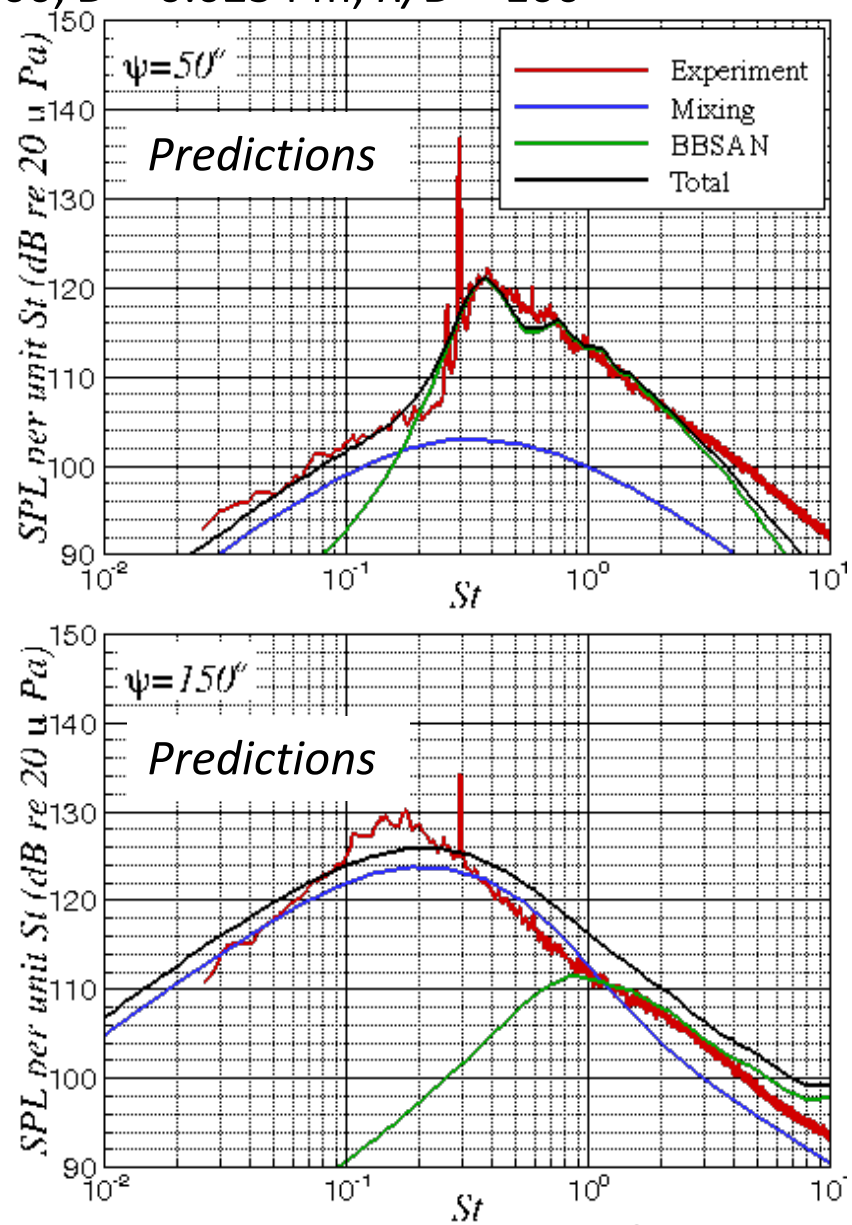
Radiation of Waves from Turbulence



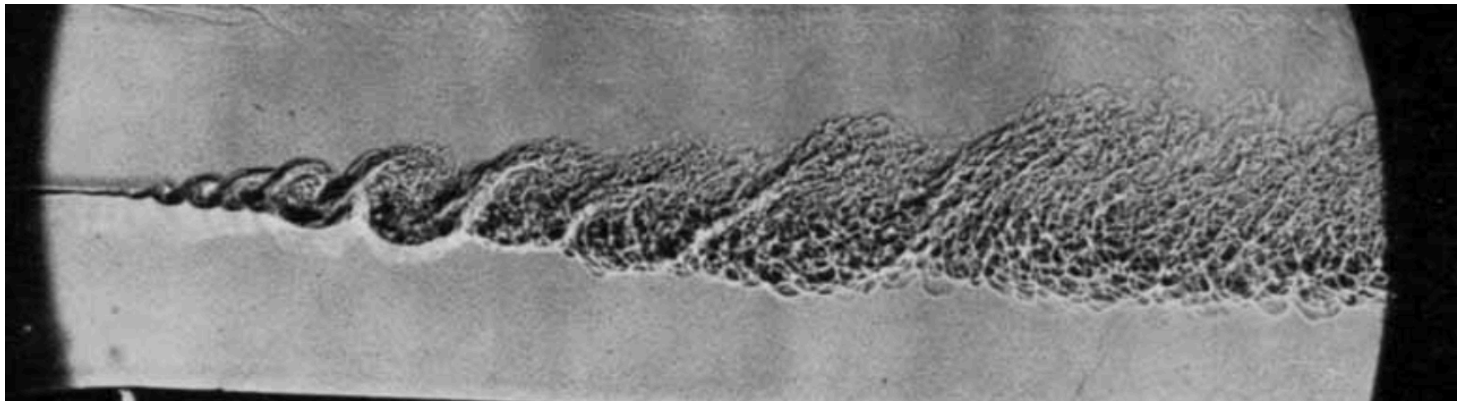
Jet Noise Spectra

M_d = 1.00, M_j = 1.50, TTR = 1.00, D = 0.0254 m, R/D = 100





Large-Scale Coherent Anisotropic Structures and Instability Waves



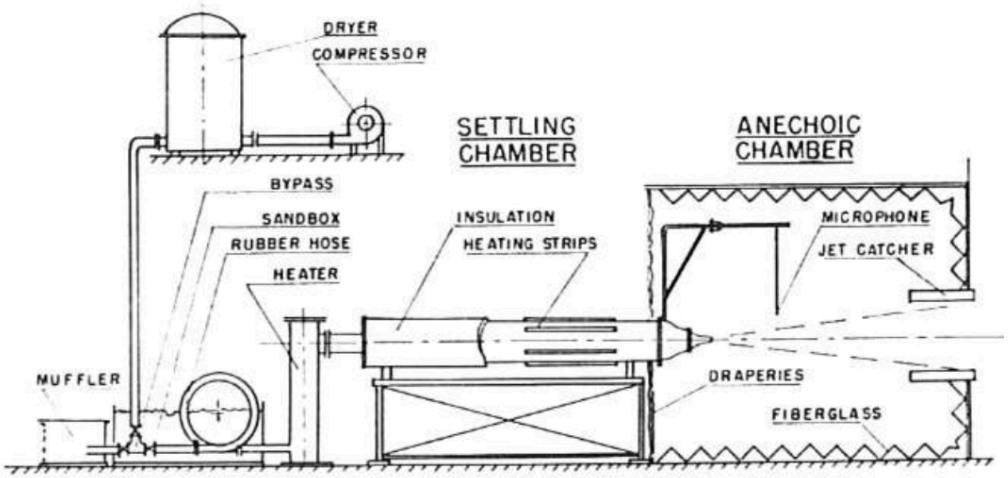
Liu JFM 1974

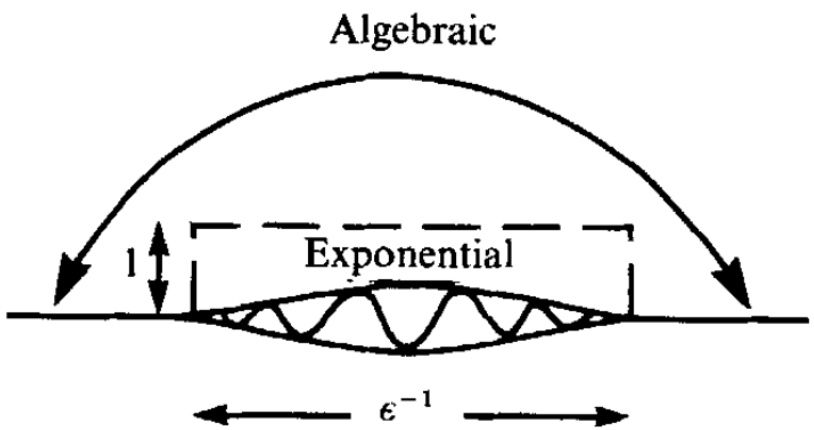
(*b*)

FIGURE 4. Large structure 120 cm (picture centre) downstream of the splitter plate; $U_1 = 100$ cm/s, flow from right to left, high-speed side at the bottom. (*a*) Injection from high-speed side. (*b*) Injection from low-speed side.

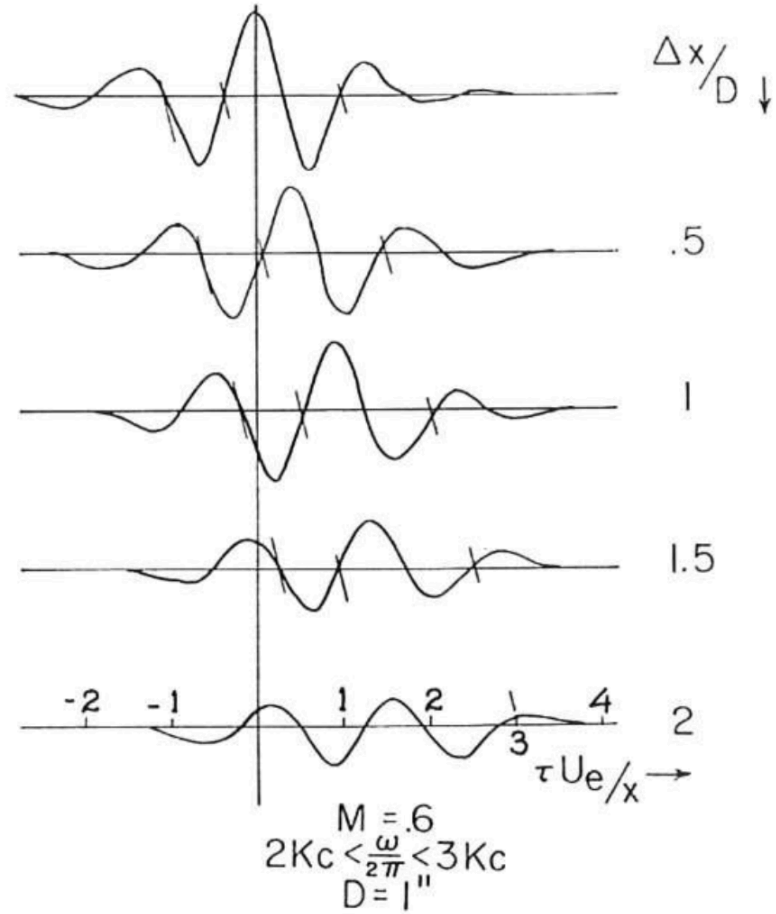
Dimotakis and Brown JFM 1976

Large-Scale Coherent Anisotropic Structures and Instability Waves



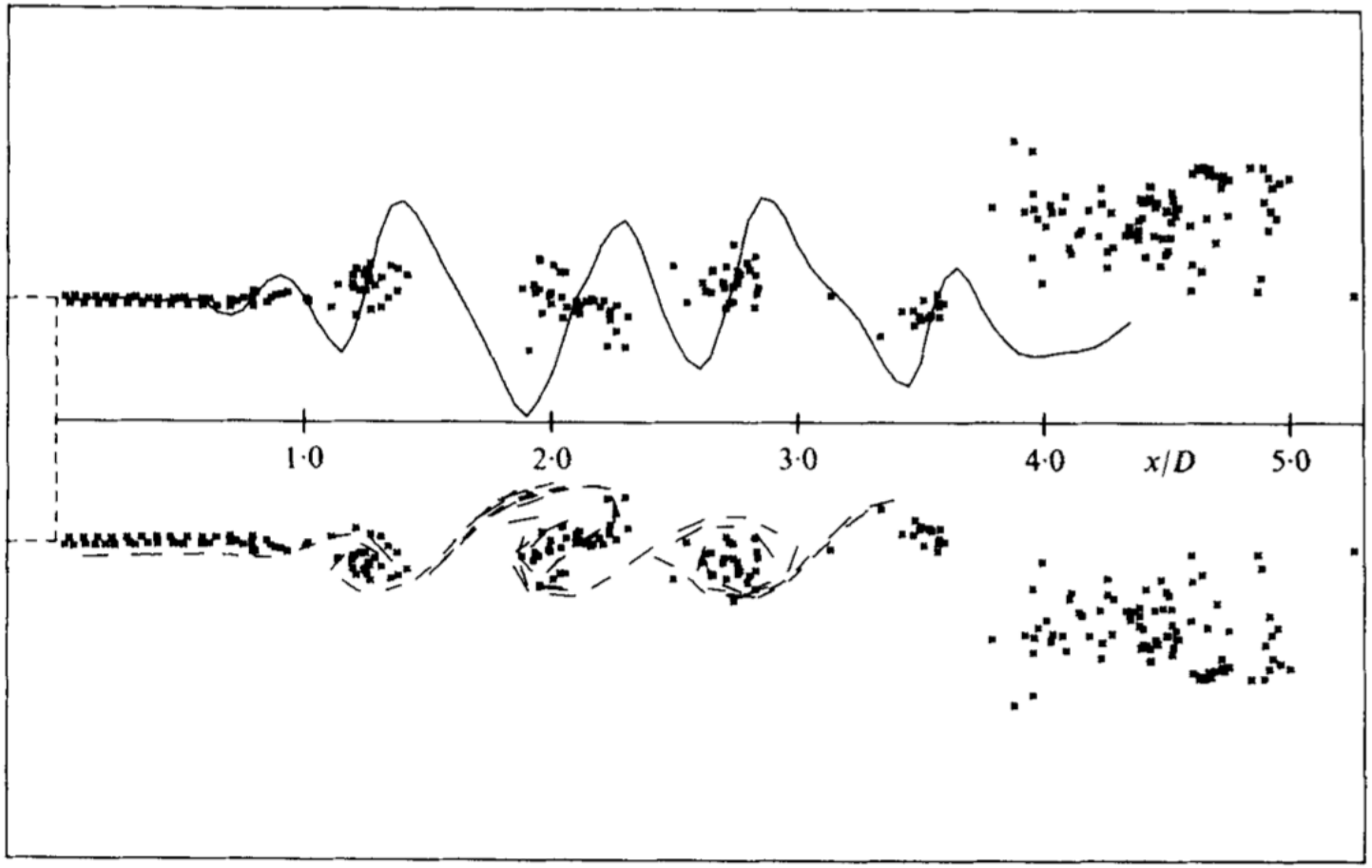


Crighton and Huerre JFM 1990



Experiments of Mollo-Christiansen 1967

Large-Scale Coherent Anisotropic Structures and Instability Waves



Unforced jet – top radial velocity trace, dye line, vortex element

Acton 1976

Large-Scale Coherent Anisotropic Structures and Instability Waves

Jordon and Colonius Annual Review Fluids 2013

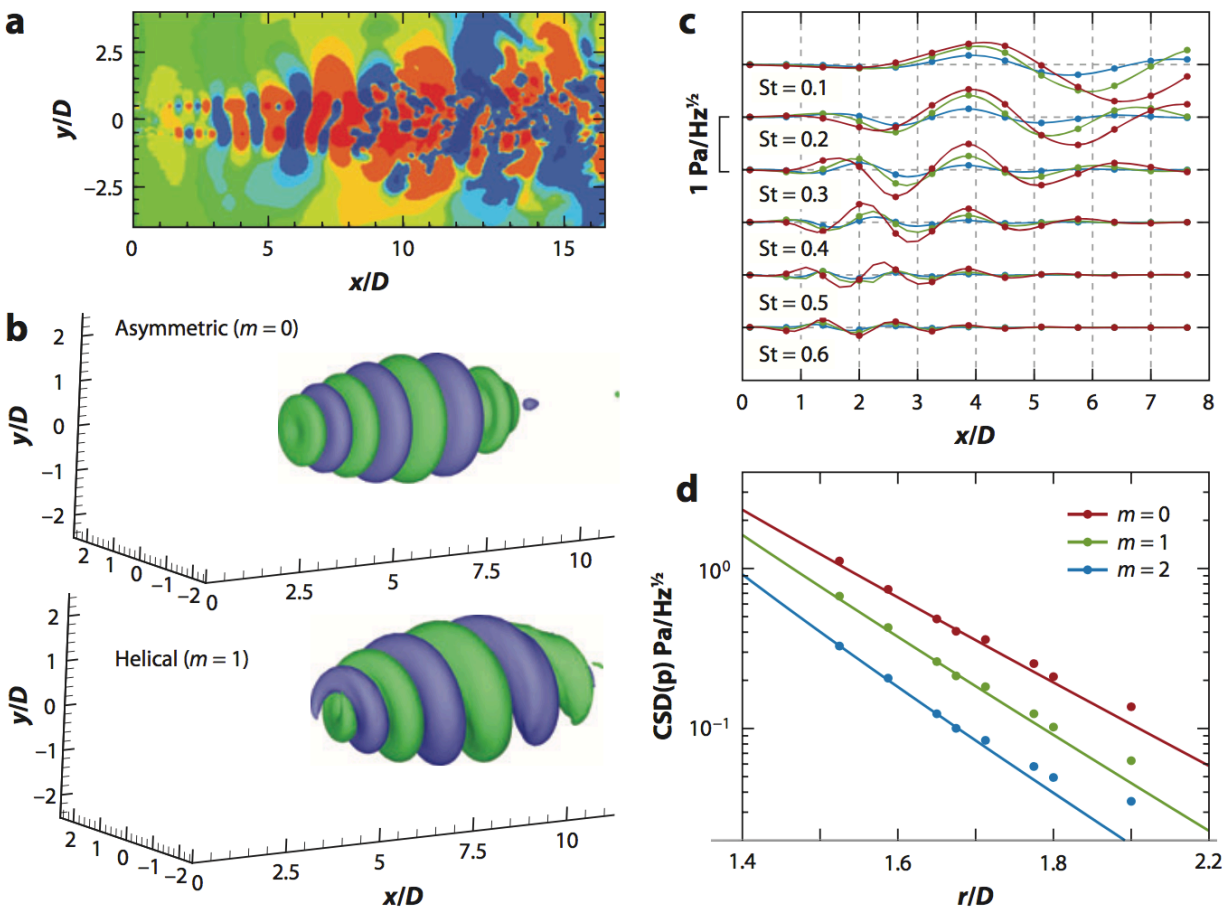


Figure 4

Wave packets: (*a*) instantaneous slice of the $M_j = 0.9$, Re $= 3{,}600$ DNS of Freund (2001); (*b*) the most energetic axisymmetric ($m = 0$) and helical ($m = 1$) pressure wave packets from the DNS; (*c*) the streamwise structure as a function of St and m based on the cross-spectral density (CSD) of the pressure from a near-field caged microphone array (Suzuki & Colonius 2006) at $M_j = 0.5$, Re $= 700{,}000$; and (*d*) radial decay at $x/D = 3.5$ from the same experiments. The lines represent the exponential radial decay associated with $J_m(i\beta r)$, where $\beta = \sqrt{\alpha^2 - (\frac{\omega}{a_\infty})^2}$, with α the measured local wave number.

A Note on the Universality: Wavenumber Spectra of Structure Functions

Velocity

$$E_u \propto c_u \epsilon^{\alpha_u} \kappa^{-5/3}$$

Kolmogorov, A. N., "The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Number," Dokl. Akad. Nauk. SSSR, Vol. 30, 1941, pp. 299–303.

Pressure

$$E_p \propto c_p \epsilon^{\alpha_p} \kappa^{-7/3}$$

Batchelor, G. K., "Pressure Fluctuations in Isotropic Turbulence," Math. Proc. Camb. Phil. Soc., Vol. 47, No. 2, 1951, pp. 359–374. doi:10.1017/S0305004100026712.

Temperature

$$E_T \propto c_T \epsilon^{\alpha_T} \kappa^{-5/3}$$

Corrsin, S., "On the Spectrum of Isotropic Temperature Fluctuations in an Isotropic Turbulence," Journal of Applied Physics, Vol. 22, No. 4, 1951, pp. 469–473. doi:10.1063/1.1699986.

Density

$$E_\rho \propto c_\rho \epsilon^{\alpha_\rho} \kappa^{-5/3}$$

Montgomery, D., Brown, M. R., and Matthaeus, W. H., "Density Fluctuation Spectra in Magnetohydrodynamic Turbulence," Journal of Geophysical Research, Vol. 92, No. A1, 1987, pp. 282–284. doi:10.1029/JA092iA01p00282.

Modeling Goals

- Create an acoustic analogy to predict noise from turbulent flows that is closed-form
- Arguments of the model
  - Large-scale anisotropic turbulence
  - Wavenumber `energy' spectra
- Validate the model through comparison with measurements
- Gain insight into the physical mechanism of sound generation

Mathematical Methods

Mathematical Outline

- Governing equations
- Decomposition
- Sources
- A statistical solution through acoustic analogy
- Modeling sources
- Final model equation

Governing Equations

Navier-Stokes equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_j}\delta_{ij} + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$\frac{\partial \rho e_o}{\partial t} + \frac{\partial \rho u_j e_o}{\partial x_j} = -\frac{\partial u_j p}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \frac{\partial u_j \tau_{ij}}{\partial x_j}$$

Decomposition of Governing Equations

Decompose Navier-Stokes equations where q is a field-variable

$$\boldsymbol{q} = \overline{q} + \tilde{q} + \tilde{\tilde{q}} + q' + q''$$

and,

$\overline{q} =$ Time-evolving or steady non-radiating base flow

$\tilde{q} =$ Anisotropic turbulent fluctuations

$\tilde{\tilde{q}} =$ Isotropic turbulent fluctuations

$q' =$ Radiating waves associated with anisotropic turbulence

$q'' =$ Radiating waves associated with isotropic turbulence

Θ_i Sources are ***exactly*** Navier-Stokes equations operators on $\underline{\boldsymbol{q}}$

Source Terms

$$\Theta_0 = -\frac{\overline{\rho}+\tilde{\rho}}{\tau} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})}{l_x} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{v}+\tilde{v})}{l_y}$$

$$\Theta_1 = -\frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})}{\tau} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})^2}{l_x} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_y} - \frac{\overline{p}+\tilde{p}}{l_x} + \frac{4\mu}{3}\frac{(\overline{u}+\tilde{u})^2}{l_x^2} + \mu\frac{(\overline{u}+\tilde{u})^2}{l_y^2} + \frac{\mu}{3}\frac{(\overline{v}+\tilde{v})^2}{l_x l_y}$$

$$\Theta_2 = -\frac{(\overline{\rho}+\tilde{\rho})(\overline{v}+\tilde{v})}{\tau} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_x} - \frac{(\overline{\rho}+\tilde{\rho})(\overline{v}+\tilde{v})^2}{l_y} - \frac{\overline{p}+\tilde{p}}{l_y} + \frac{4\mu}{3}\frac{(\overline{v}+\tilde{v})^2}{l_y^2} + \mu\frac{(\overline{v}+\tilde{v})^2}{l_y^2} + \frac{\mu}{3}\frac{(\overline{u}+\tilde{u})^2}{l_x l_y}$$

$$\begin{aligned}\Theta_3 = &-\frac{\overline{p}+\tilde{p}}{\tau} - \frac{\gamma-1}{2}\left(\frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})^2}{\tau} + \frac{(\overline{\rho}+\tilde{\rho})(\overline{v}+\tilde{v})^2}{\tau}\right) - \gamma\left(\frac{(\overline{u}+\tilde{u})(\overline{p}+\tilde{p})}{l_x} + \frac{(\overline{v}+\tilde{v})(\overline{p}+\tilde{p})}{l_y}\right)\\
&-\frac{\gamma-1}{2}\left(\frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})^3}{l_x} + \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})^2}{l_x} + \frac{(\overline{\rho}+\tilde{\rho})(\overline{u}+\tilde{u})^2(\overline{v}+\tilde{v})}{l_y} + \frac{(\overline{\rho}+\tilde{\rho})(\overline{v}+\tilde{v})^3}{l_y}\right)\\
&+(\gamma-1)\frac{c_p\mu}{Pr}\left(\frac{\overline{T}+\tilde{T}}{l_x^2} + \frac{\overline{T}+\tilde{T}}{l_y^2}\right) + (\gamma-1)\mu\left(\frac{2(\overline{u}+\tilde{u})^2}{l_x^2} + \frac{(\overline{u}+\tilde{u})^2}{l_x l_y} + \frac{(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_x^2}\right)\\
&+(\gamma-1)\mu\left(\frac{2(\overline{v}+\tilde{v})^2}{l_y^2} + \frac{(\overline{v}+\tilde{v})^2}{l_x l_y} + \frac{(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_y^2}\right)\\
&-\frac{2\mu}{3}\left(\frac{(\overline{u}+\tilde{u})^2}{l_x^2} + \frac{(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_x l_y}\right) - \frac{2\mu}{3}\left(\frac{(\overline{v}+\tilde{v})^2}{l_y^2} + \frac{(\overline{u}+\tilde{u})(\overline{v}+\tilde{v})}{l_x l_y}\right)\end{aligned}$$

Seek a Solution

Takes the form

$$q_k^{\perp}(\boldsymbol{x},t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{n=0}^{3} \boldsymbol{q}_{g,k}^{\perp n}(\boldsymbol{x},t;\boldsymbol{y},\tau)\,\Theta_n(\boldsymbol{y},\tau)\,d\tau d\boldsymbol{y}.$$

Satisfies the vector Green's function

$$\frac{\partial \rho_g^{\perp n}}{\partial t} + \frac{\partial}{\partial x_j}\left[\rho_g^{\perp n}\underline{u}_j + u_{j,g}^{\perp n}\underline{\rho}\right] = \delta(\boldsymbol{x}-\boldsymbol{y})\,\delta(t-\tau)\,\delta_{0n}$$

$$\frac{\partial}{\partial t}\left[\rho_g^{\perp n}\underline{u}_i + u_{i,g}^{\perp n}\underline{\rho}\right] + \frac{\partial}{\partial x_j}\left[\rho_g^{\perp n}\underline{u}_i\underline{u}_j + u_{i,g}^{\perp n}\underline{\rho}\underline{u}_j + u_{j,g}^{\perp n}\underline{\rho}\underline{u}_i\right] - \overline{\mu}\frac{\partial}{\partial x_j}\left[\frac{\partial u_i^{\perp n}}{\partial x_j} + \frac{\partial u_j^{\perp n}}{\partial x_i}\right]$$
$$+\frac{2}{3}\overline{\mu}\frac{\partial^2 u_k^{\perp n}}{\partial x_j \partial x_k} + \frac{\partial p^{\perp n}}{\partial x_j}\delta_{ij} = \delta(\boldsymbol{x}-\boldsymbol{y})\,\delta(t-\tau)\,\delta_{in},$$

$$\frac{\partial p^{\perp n}}{\partial t} + \frac{\gamma-1}{2}\frac{\partial}{\partial t}\left[\rho^{\perp n}\underline{u}_k^2 + 2u_k^{\perp n}\underline{\rho}\underline{u}_k\right] + \gamma\frac{\partial}{\partial x_j}\left[p^{\perp n}\underline{u}_j + u_j^{\perp n}\underline{p}\right]$$
$$+\frac{\gamma-1}{2}\frac{\partial}{\partial x_j}\left[\rho^{\perp n}\underline{u}_j\underline{u}_k^2 + u_j^{\perp n}\underline{\rho}\underline{u}_k^2 + 2u_k^{\perp n}\underline{\rho}\underline{u}_k\right] - \frac{\partial}{\partial x_j}\frac{c_p\mu}{\mathcal{P}rR}\frac{\partial}{\partial x_j}\left[\frac{\tilde{p}^n}{\tilde{\rho}^n} + \frac{\tilde{\tilde{p}}^n}{\tilde{\tilde{\rho}}^n}\right]$$
$$-(\gamma-1)\,\overline{\mu}\frac{\partial}{\partial x_j}\left[\underline{u}_i\left(\frac{\partial u_i^{\perp n}}{\partial x_j} + \frac{u_j^{\perp n}}{\partial x_i}\right) + u_i^{\perp n}\left(\frac{\underline{u}_i}{\partial x_j} + \frac{\underline{u}_j}{\partial x_i}\right)\right]$$
$$+\frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu(\underline{u}_i)\frac{\partial u_k^{\perp n}}{\partial x_k} + \mu\left(u_i^{\perp n}\right)\frac{\partial \underline{u}_k}{\partial x_k}\right]$$
$$= \delta(\boldsymbol{x}-\boldsymbol{y})\,\delta(t-\tau)\,\delta_{3n}.$$

Seek a Predictable Statistical Solution

Fourier transform of the solution

$$\hat{q}_k^{\perp}\left(\boldsymbol{x},\omega\right) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \sum_{j=0}^{4} q_{g,k}^{\perp j}\left(\boldsymbol{x},t;\boldsymbol{y},\tau\right)\Theta_j\left(\boldsymbol{y},\tau\right)\exp\left[-i\omega t\right] dt d\tau d\boldsymbol{y}.$$

Represents the Fourier transform of $\boldsymbol{q}$ and is complicated by the fact that it contains both the retarded time, τ, and frequency, ω

$$S_k^{\perp}\left(\boldsymbol{x},\omega\right) = \int_{-\infty}^{\infty} \left\langle q_k^{\perp}\left(\boldsymbol{x},t\right) q_k^{\perp}\left(\boldsymbol{x},t+\tau^{\dagger}\right)\right\rangle \exp\left[i\omega\tau^{\dagger}\right] d\tau^{\dagger},$$

A General Model for Spectral Density

We define $R_{m,n}$ $(\boldsymbol{y}, \eta, \tau)$ as the two-point space-time cross-correlation of the equivalent source

$$S_k^{\perp}(\boldsymbol{x}, \omega) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \sum_{m=0}^{3} \sum_{n=0}^{3} \hat{q}_{g,k}^{*\perp m}(\boldsymbol{x}; \boldsymbol{y}, \omega)\, \hat{q}_{g,k}^{\perp n}(\boldsymbol{x}; \boldsymbol{y} + \boldsymbol{\eta}, \omega)\, R_{m,n}^{\perp}(\boldsymbol{y}, \boldsymbol{\eta}, \tau)\, d\tau d\boldsymbol{\eta} d\boldsymbol{y}.$$

where

$$R_{m,n}^{\perp}(\boldsymbol{y}, \boldsymbol{\eta}, \tau) = \langle \Theta_m(\boldsymbol{y}, \tau)\, \Theta_n(\boldsymbol{y} + \boldsymbol{\eta}, \tau + \Delta\tau) \rangle = \int_{-\infty}^{\infty} \Theta_m(\boldsymbol{y}, \tau)\, \Theta_n(\boldsymbol{y} + \boldsymbol{\eta}, \tau + \Delta\tau)\, d\Delta\tau.$$

Model the source terms and assume separable

$$R_{m,n}^{\perp}(\boldsymbol{y}, \boldsymbol{\eta}, \tau) \approx \left\{ R_{m,n}^{\perp} \right\} R$$

Modeling Two-Point Cross-Correlation

Normalized two-point cross-correlation

$$\boldsymbol{R} = \int_{-\infty}^{\infty} \mathcal{P}_1(y_1, \Delta_1, \Delta_2, t)\mathcal{P}_2(y, \delta_1, \delta_2, \eta_1, t, \tau)dt$$

is a function of two large-scale instability waves of the form

$$\mathcal{P}_1(y_1, \Delta_1, \Delta_2, t) = \exp\left[-\frac{\Delta_{(2)}^2}{l_{y,(1)}^2}\right] \exp\left[-\frac{t^2}{\tau_{c,(1)}^2}\right]$$
$$\times \exp\left[-\frac{(\Delta_{(1)} + y_{(1)})^2}{l_{x,1}^2}\right] \exp\left[i(\kappa_{(1)}(\Delta_{(1)} + y_{(1)}) - \kappa_{(1)} t u_{c,(1)})\right]$$

Models of Source Terms

One of 16 source terms

$$
\begin{aligned}
\Theta_1\Theta_3 = \Bigg(& \frac{4\mu(\breve{u}_{(1)}+\overline{u}_{(1)})^2}{3l_{x,(1)}^2} + \frac{\mu(\breve{v}_{(1)}+\overline{v}_{(1)})^2}{3l_{x,(1)}l_{y,(1)}} - \frac{\breve{p}_{(1)}+\overline{p}_{(1)}}{l_{x,(1)}} - \frac{(\breve{\rho}_{(1)}+\overline{\rho}_{(1)})(\breve{u}_{(1)}+\overline{u}_{(1)})^2}{l_{x,(1)}} \\
& + \frac{\mu(\breve{u}_{(1)}+\overline{u}_{(1)})^2}{l_{y,(1)}^2} - \frac{(\breve{\rho}_{(1)}+\overline{\rho}_{(1)})(\breve{u}_{(1)}+\overline{u}_{(1)})(\breve{v}_{(1)}+\overline{v}_{(1)})}{l_{y,(1)}} - \frac{(\breve{\rho}_{(1)}+\overline{\rho}_{(1)})(\breve{u}_{(1)}+\overline{u}_{(1)})}{\tau_{(1)}} \Bigg) \\
& \times \Bigg(\frac{(\gamma-1)c_p\mu\left(\frac{\breve{T}_{(2)}+\overline{T}_{(2)}}{l_{x,(2)}^2} + \frac{\breve{T}_{(2)}+\overline{T}_{(2)}}{l_{y,(2)}^2}\right)}{\mathcal{P}_r} + (\gamma-1)\mu\Bigg(\frac{(\breve{u}_{(2)}+\overline{u}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{x,(2)}^2} \\
& + \frac{2(\breve{u}_{(2)}+\overline{u}_{(2)})^2}{l_{x,(2)}^2} + \frac{(\breve{u}_{(2)}+\overline{u}_{(2)})^2}{l_{x,(2)}l_{y,(2)}} \Bigg) - \frac{2}{3}\mu\left(\frac{(\breve{u}_{(2)}+\overline{u}_{(2)})^2}{l_{x,(2)}^2} + \frac{(\breve{u}_{(2)}+\overline{u}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{x,(2)}l_{y,(2)}} \right) \\
& + (\gamma-1)\mu\left(\frac{(\breve{v}_{(2)}+\overline{v}_{(2)})^2}{l_{x,(2)}l_{y,(2)}} + \frac{(\breve{u}_{(2)}+\overline{u}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{y,(2)}^2} + \frac{2(\breve{v}_{(2)}+\overline{v}_{(2)})^2}{l_{y,(2)}^2} \right) \\
& - \frac{2}{3}\mu\left(\frac{(\breve{u}_{(2)}+\overline{u}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{x,(2)}l_{y,(2)}} + \frac{(\breve{v}_{(2)}+\overline{v}_{(2)})^2}{l_{y,(2)}^2} \right) \\
& - \gamma\left(\frac{(\breve{p}_{(2)}+\overline{p}_{(2)})(\breve{u}_{(2)}+\overline{u}_{(2)})}{l_{x,(2)}} + \frac{(\breve{p}_{(2)}+\overline{p}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{y,(2)}} \right) \\
& - \frac{1}{2}(\gamma-1)\Bigg(\frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{u}_{(2)}+\overline{u}_{(2)})^3}{l_{x,(2)}} + \frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{u}_{(2)}+\overline{u}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})^2}{l_{x,(2)}} \\
& + \frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{u}_{(2)}+\overline{u}_{(2)})^2(\breve{v}_{(2)}+\overline{v}_{(2)})}{l_{y,(2)}} + \frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})^3}{l_{y,(2)}} \Bigg) - \frac{\breve{p}_{(2)}+\overline{p}_{(2)}}{\tau_{(2)}} \\
& - \frac{1}{2}(\gamma-1)\left(\frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{u}_{(2)}+\overline{u}_{(2)})^2}{\tau_{(2)}} + \frac{(\breve{\rho}_{(2)}+\overline{\rho}_{(2)})(\breve{v}_{(2)}+\overline{v}_{(2)})^2}{\tau_{(2)}} \right)\Bigg). \qquad (38)
\end{aligned}
$$

$$\breve{u}_i(\boldsymbol{y},\tau) = \sqrt{\frac{2}{3}}\left(\int_{\kappa_1}^{\kappa_2} E_u(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}$$

$$\breve{p}(\boldsymbol{y},\tau) = \left(\int_{\kappa_1}^{\kappa_2} E_p(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}$$

$$\breve{\rho}(\boldsymbol{y},\tau) = \left(\int_{\kappa_1}^{\kappa_2} E_\rho(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}$$

$$\breve{T}(\boldsymbol{y},\tau) = \left(\int_{\kappa_1}^{\kappa_2} E_T(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}$$

Final Aeroacoustic Equation

We set k = 4 for pressure and find

$$S_k^{\perp}(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{m=0}^{3} \sum_{n=0}^{3} \hat{q}_{g,k}^{*\perp m}(\boldsymbol{x};\boldsymbol{y},\omega)\, \hat{q}_{g,k}^{\perp n}(\boldsymbol{x};\boldsymbol{y}+\boldsymbol{\eta},\omega)$$

$$\times \pi \tau_{(1)} \tau_{(2)} \exp\left[-\frac{1}{4}\kappa^{(1)} \tau_{(1)}^2 u_{c,(1)}^2 + i\kappa_{(1)}(\Delta_1 + y_1) - \frac{1}{4}\kappa_{(2)}^2 \tau_{(2)}^2 u_{c,(2)}^2 \right.$$

$$\left. + i\kappa_{(1)}(\delta_2 + \eta_1 + y_1) - \frac{(\Delta_1 + y_1)^2}{l_{x,(1)}^2} - \frac{(\delta 1 + \eta_1 + y_1)^2}{l_{x,(2)}^2} - \frac{\Delta_2^2}{l_{y,(1)}^2} - \frac{\delta_2^2}{l_{y,(2)}^2} \right]$$

$$\times \left\{ R_{m,n}^{\perp}(\boldsymbol{y},\boldsymbol{\eta},\tau) \right\} d\eta dy$$

We are evaluating this equation with a Python program in conjunction with RANS and/or LES solutions

Working to further simplify for the 2D shear layer case

Preliminary Results

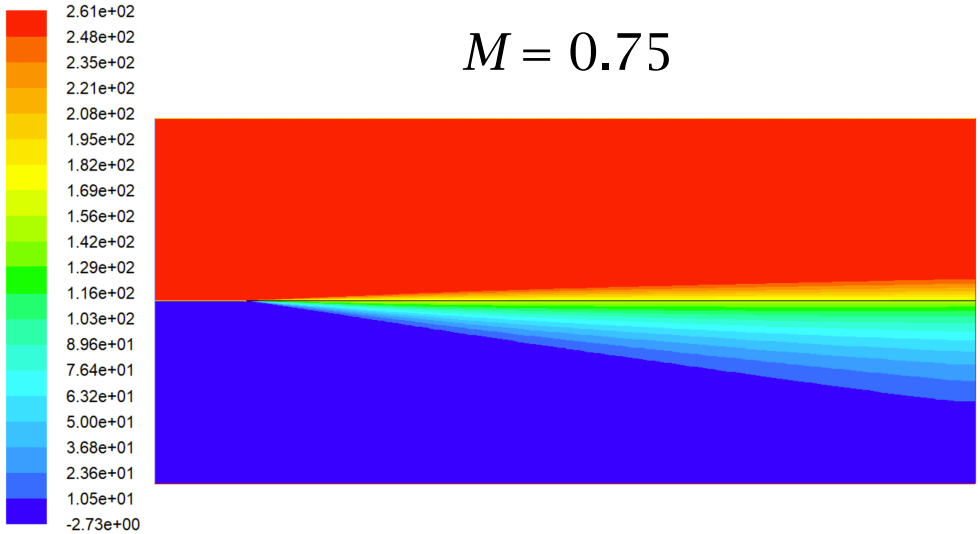
Overview

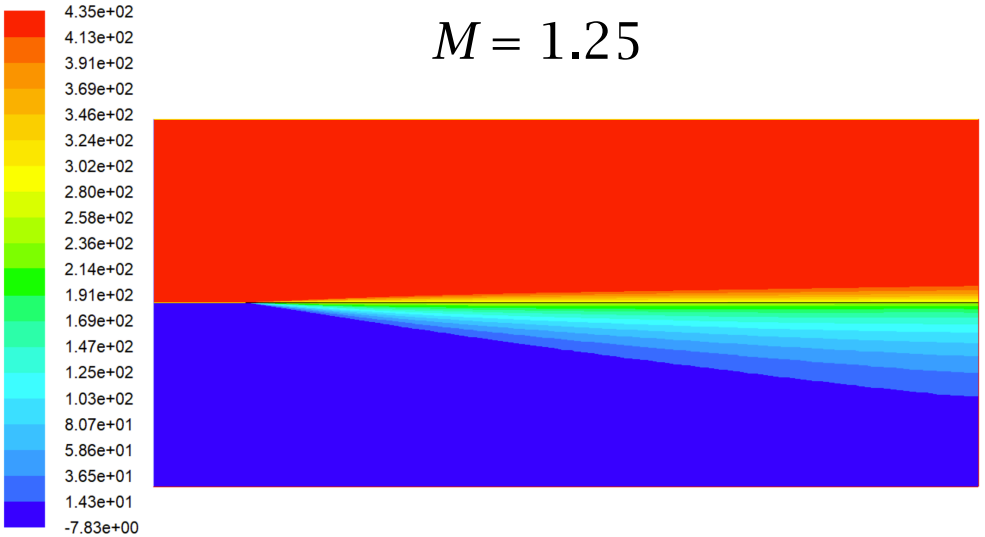
- Three shear layer Mach numbers 0.75, 1.25, and 1.75
- Select RANS and LES results
- Example instability wave model
- Acoustic Predictions

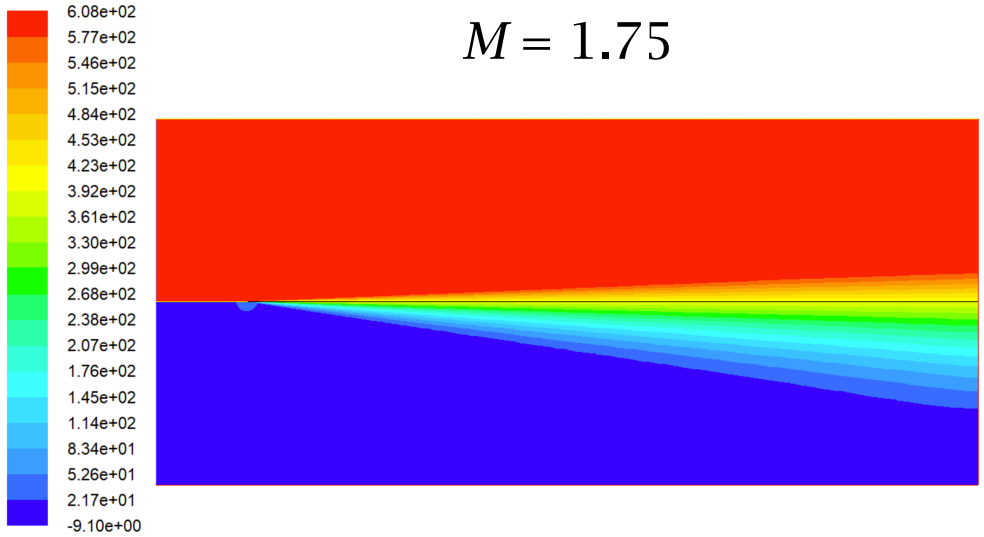
Steady RANS Contours of u

Contours of u

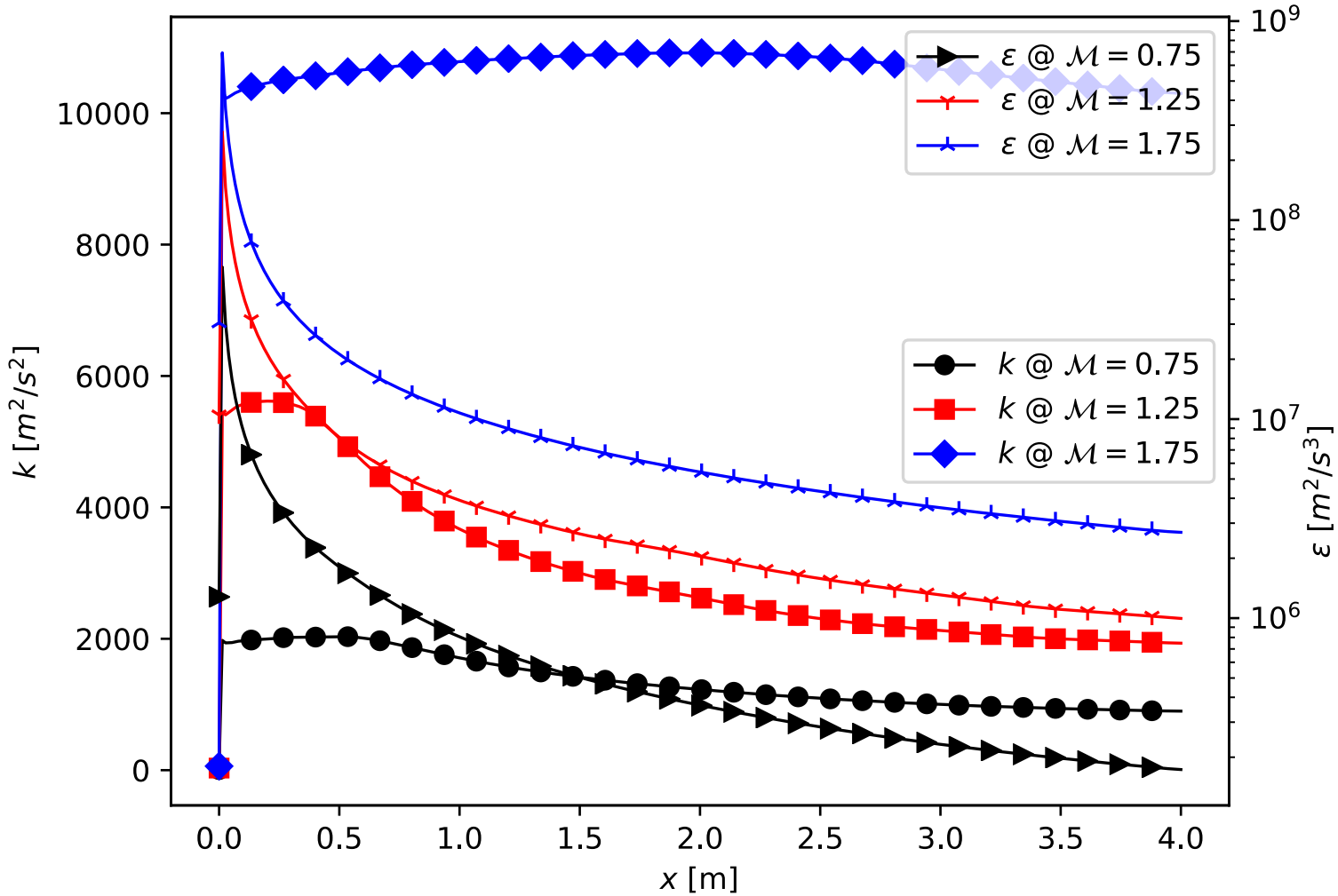
Fluent Simulations



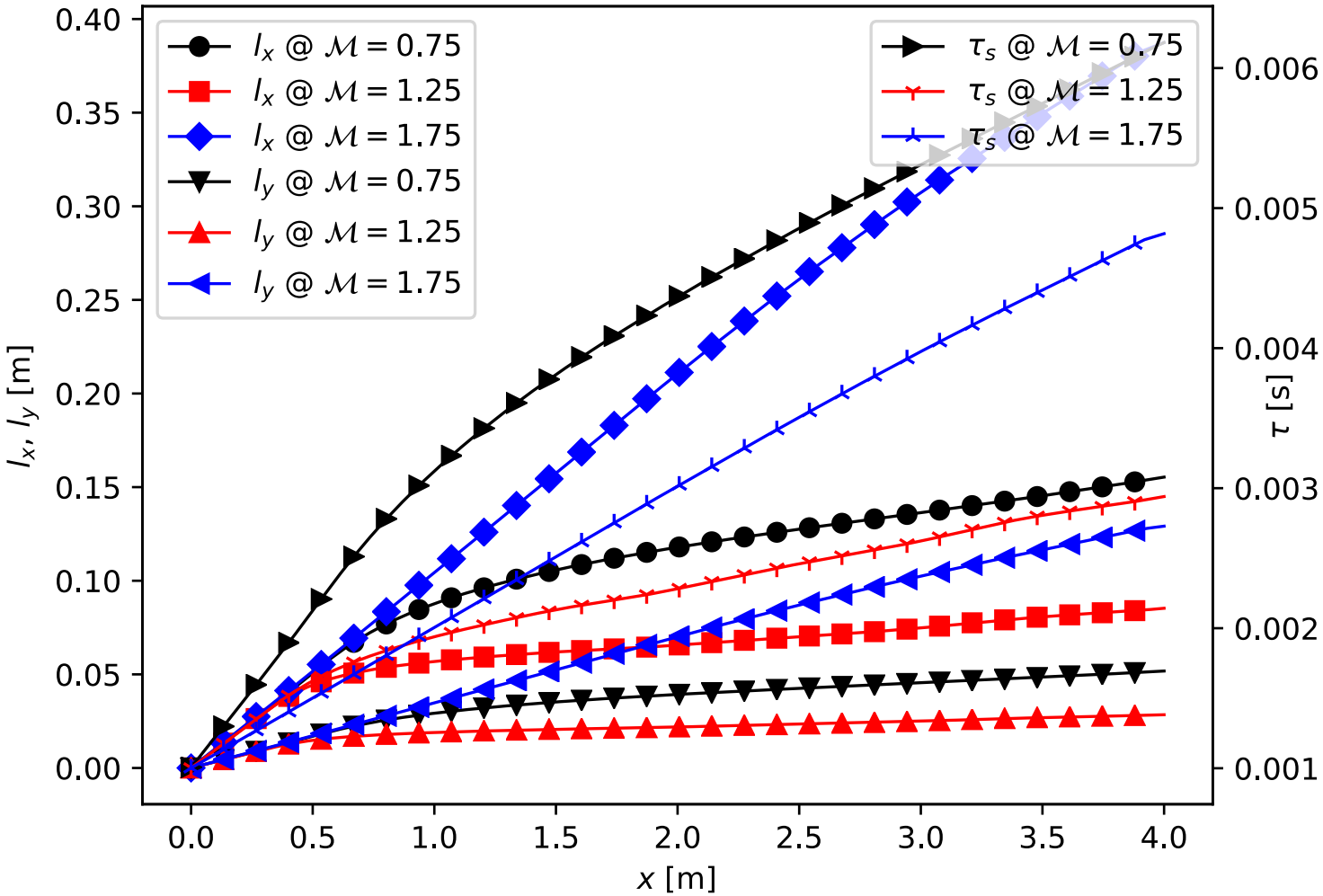




Steady RANS Model

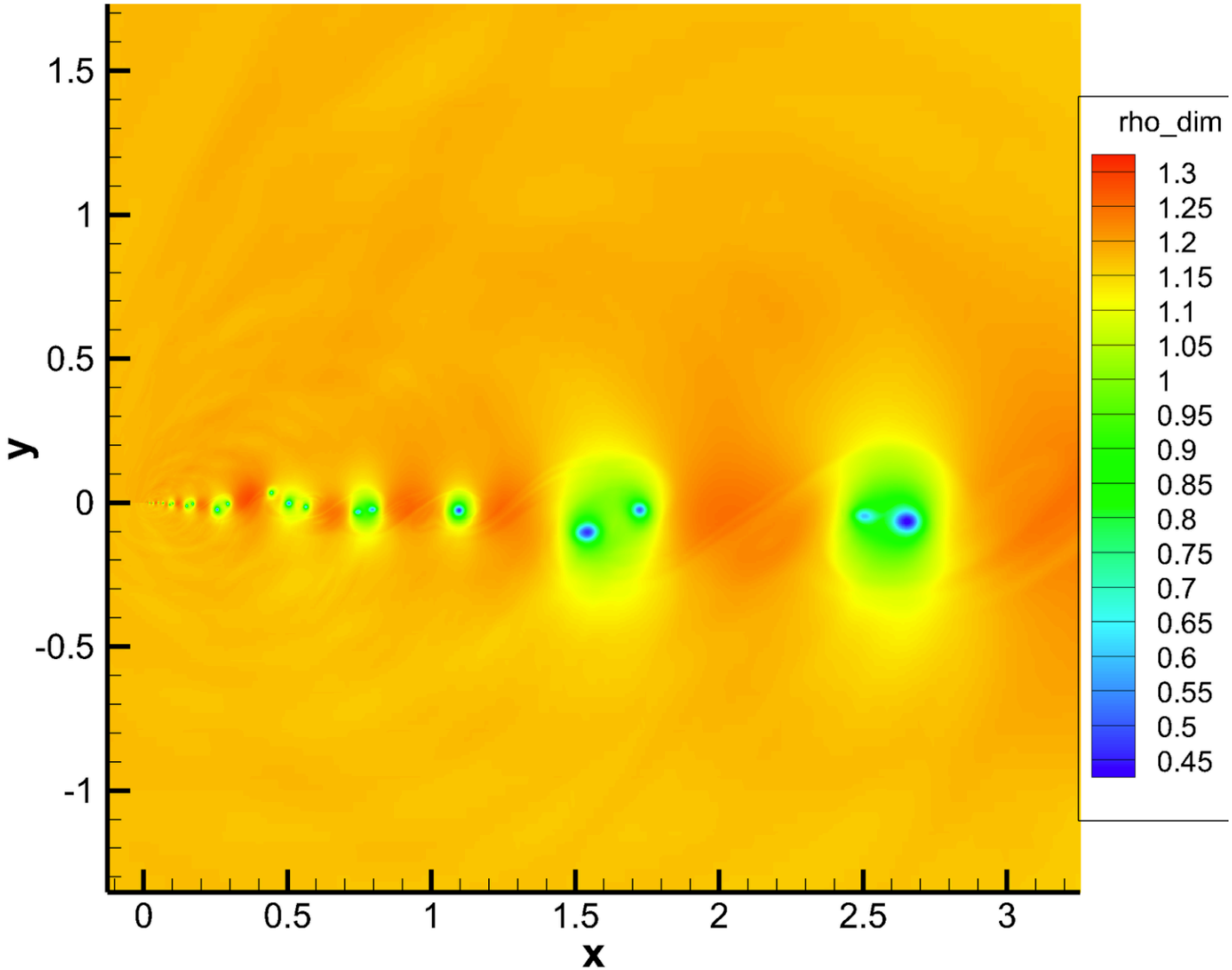


Steady RANS Model

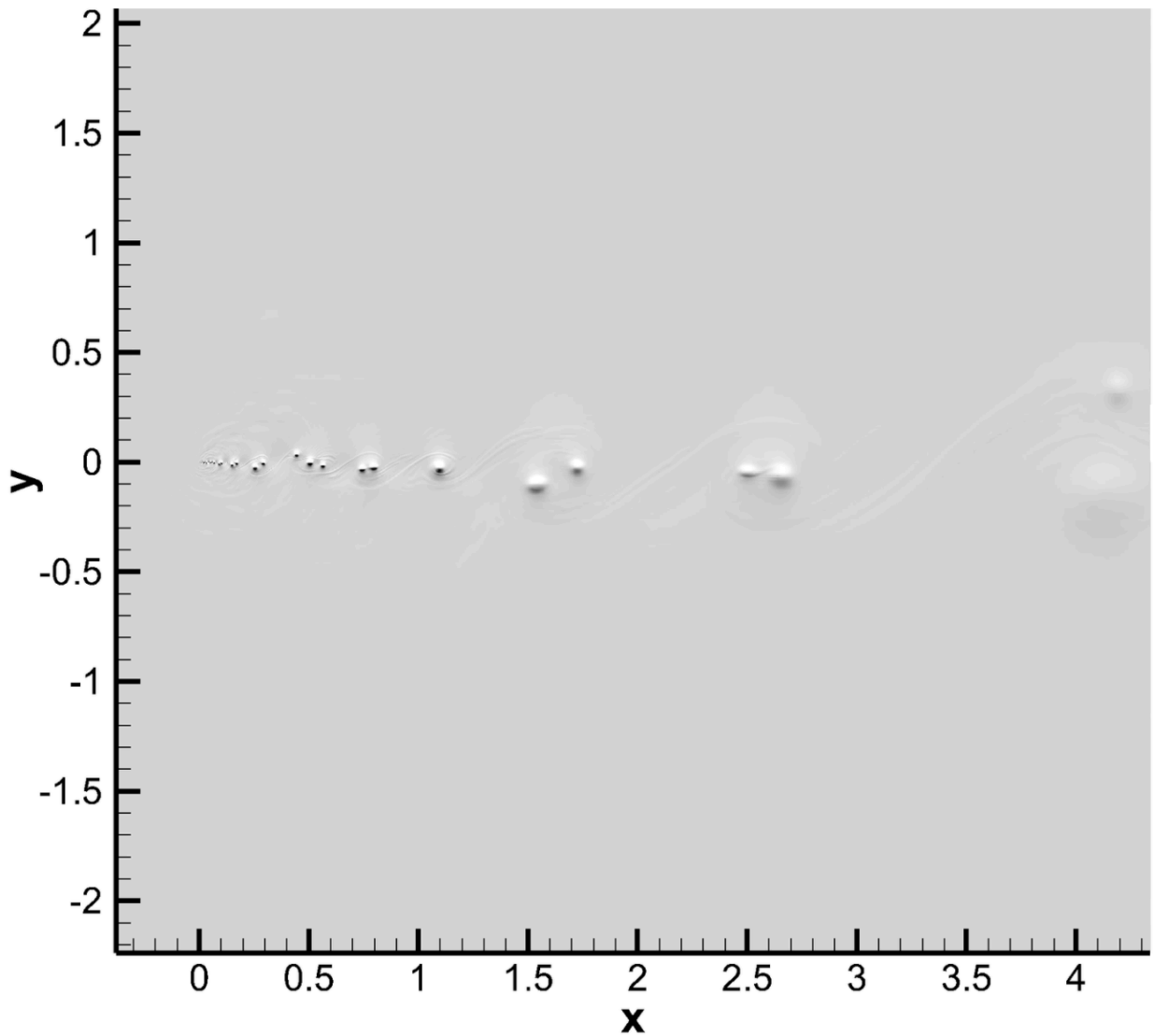


Large-Eddy Simulation

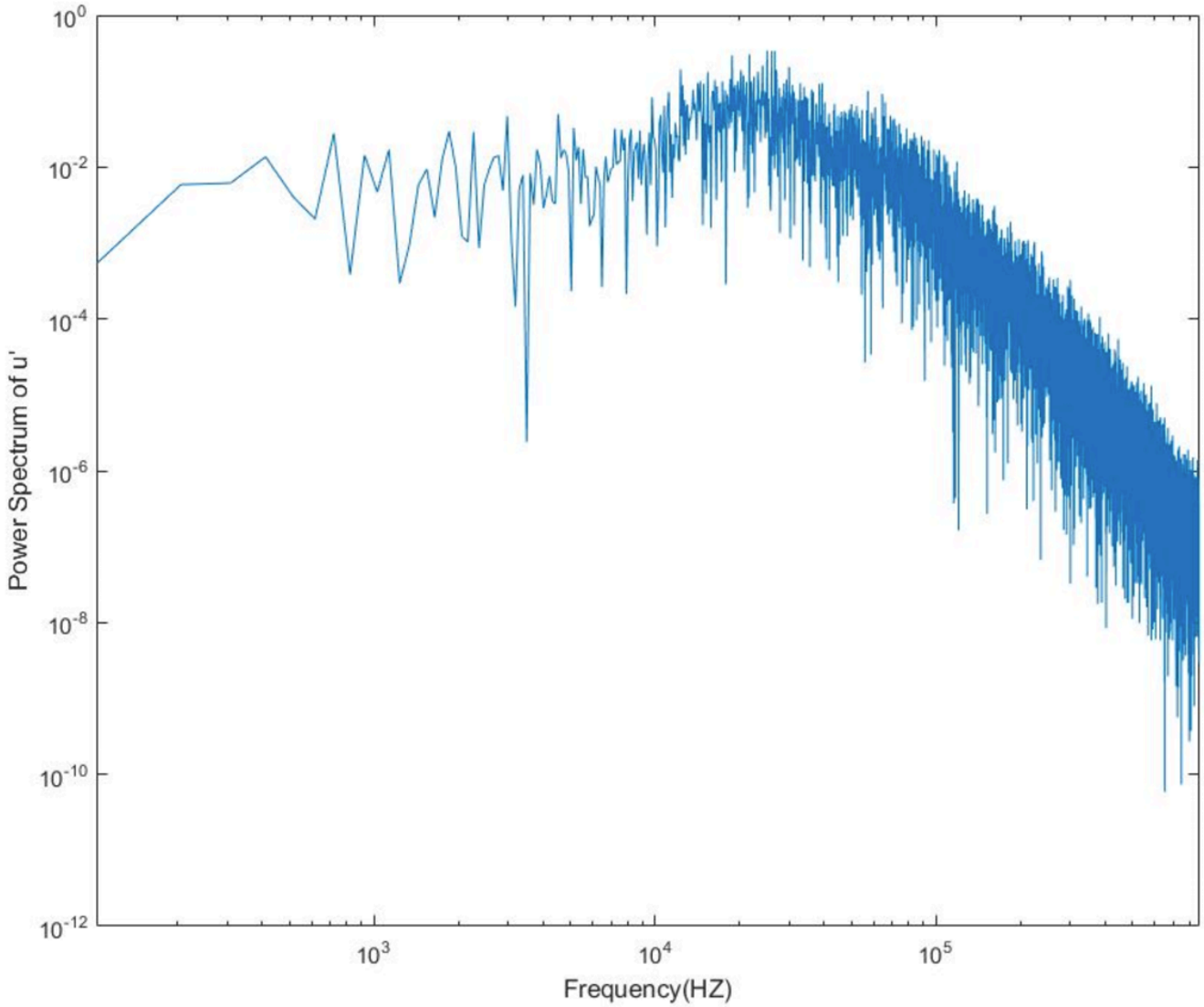
2D Shear layer at Mach 0.75



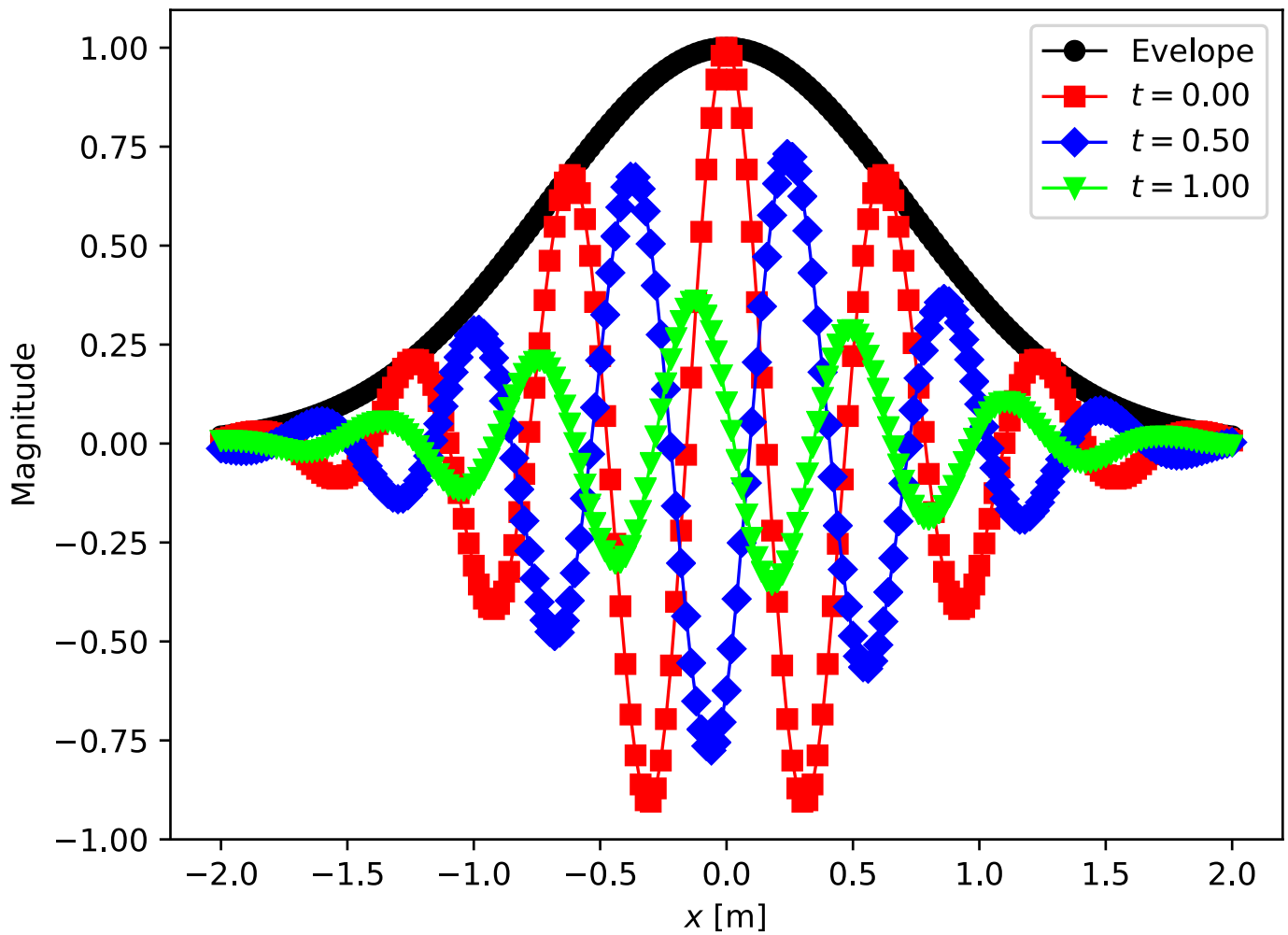
Schlieren at $M = 0.75$



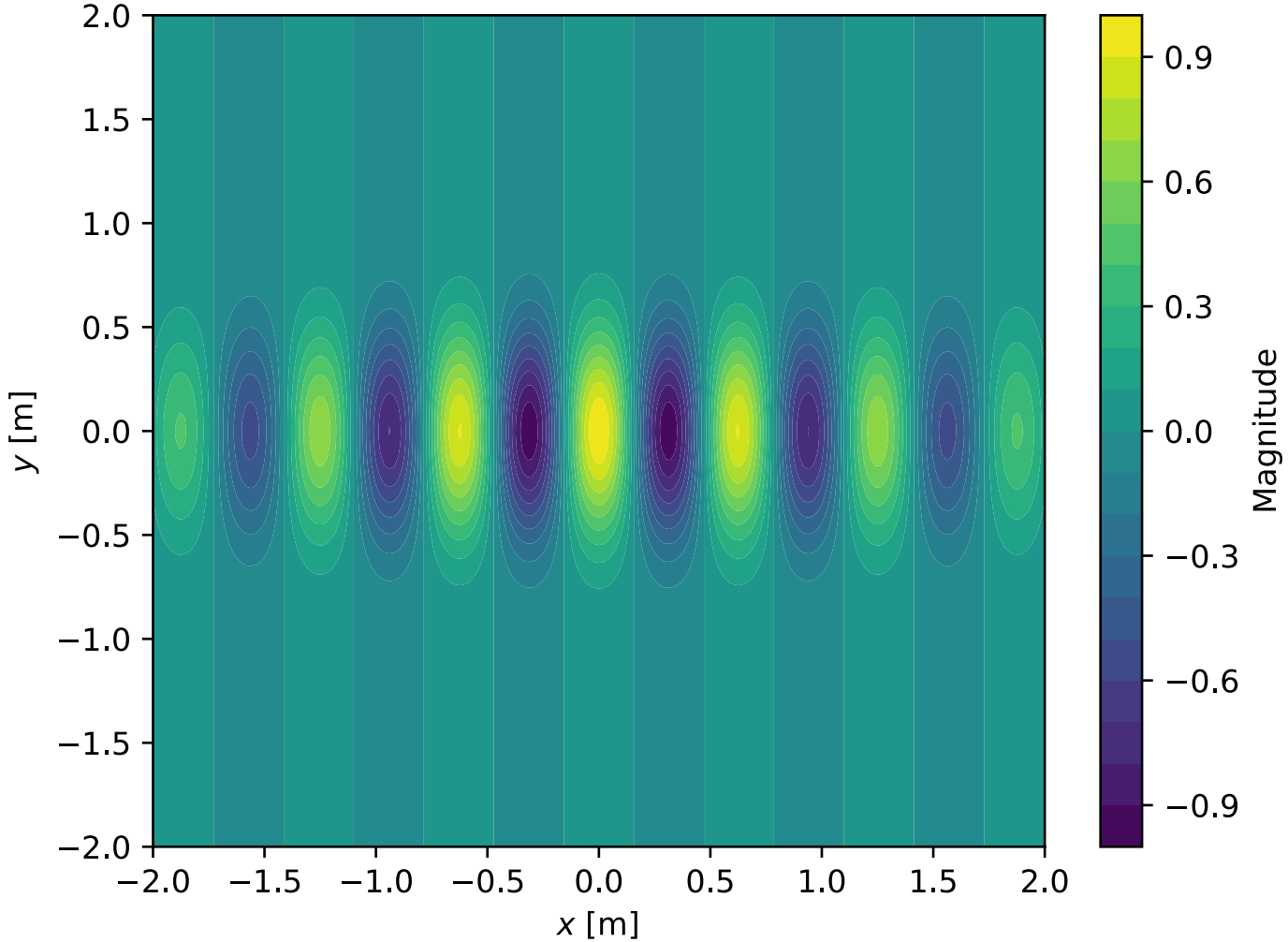
Large-Eddy Simulation E_u Power Spectrum



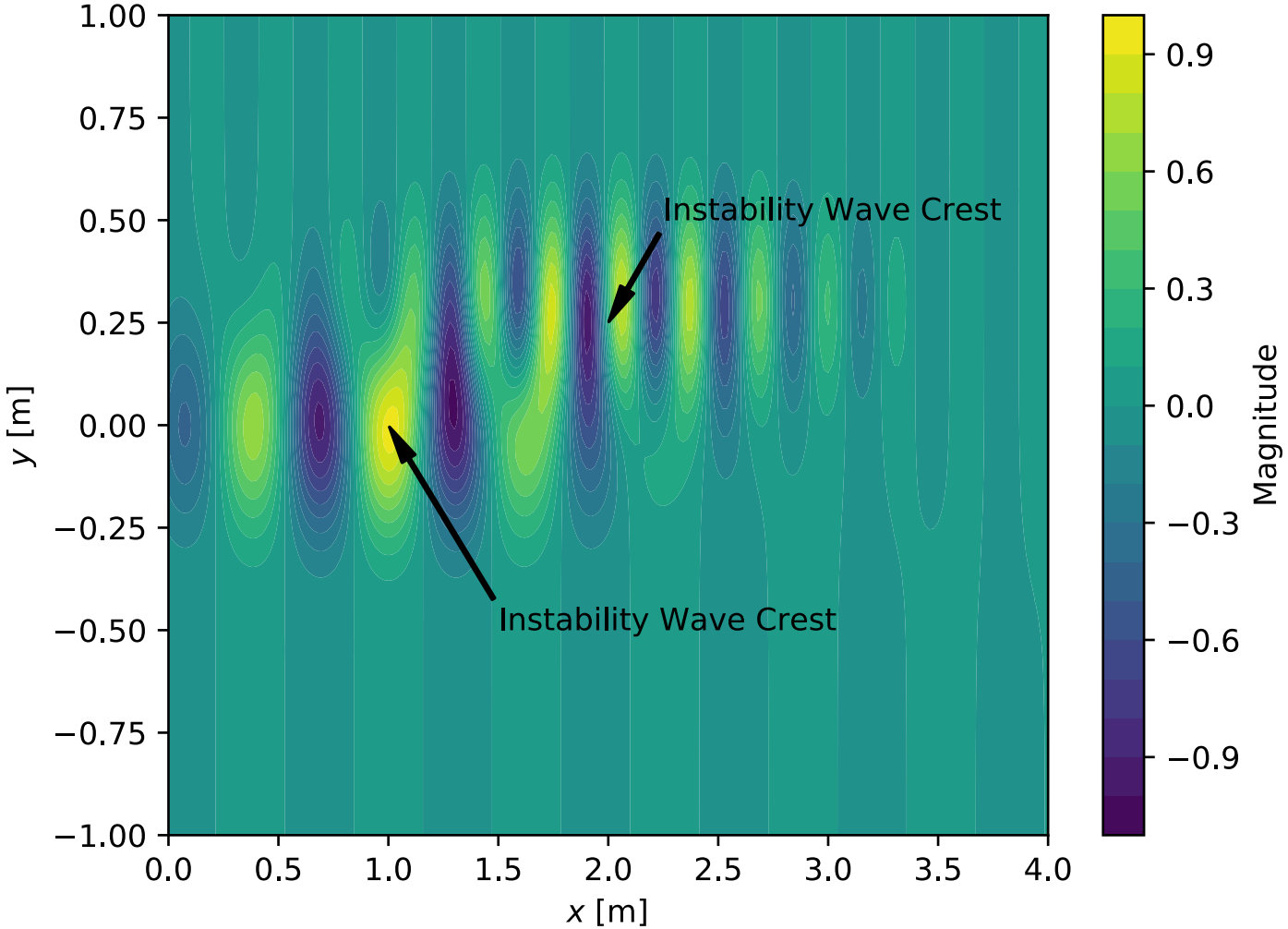
Instability Wave Evolution



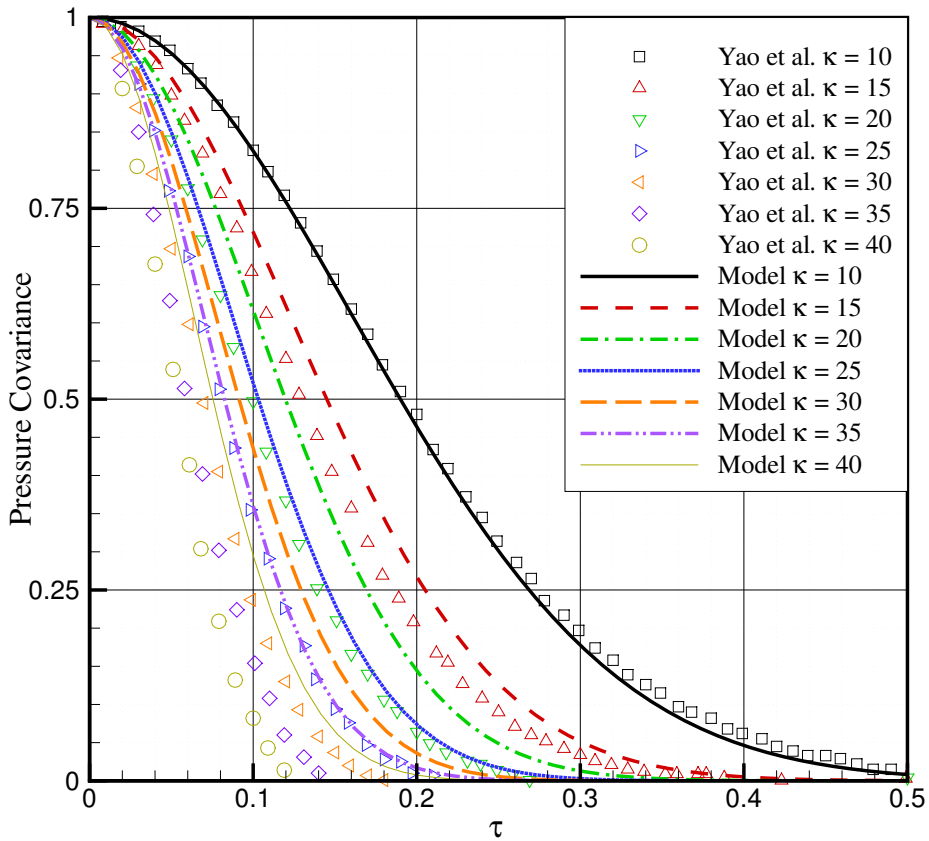
Instability Wave

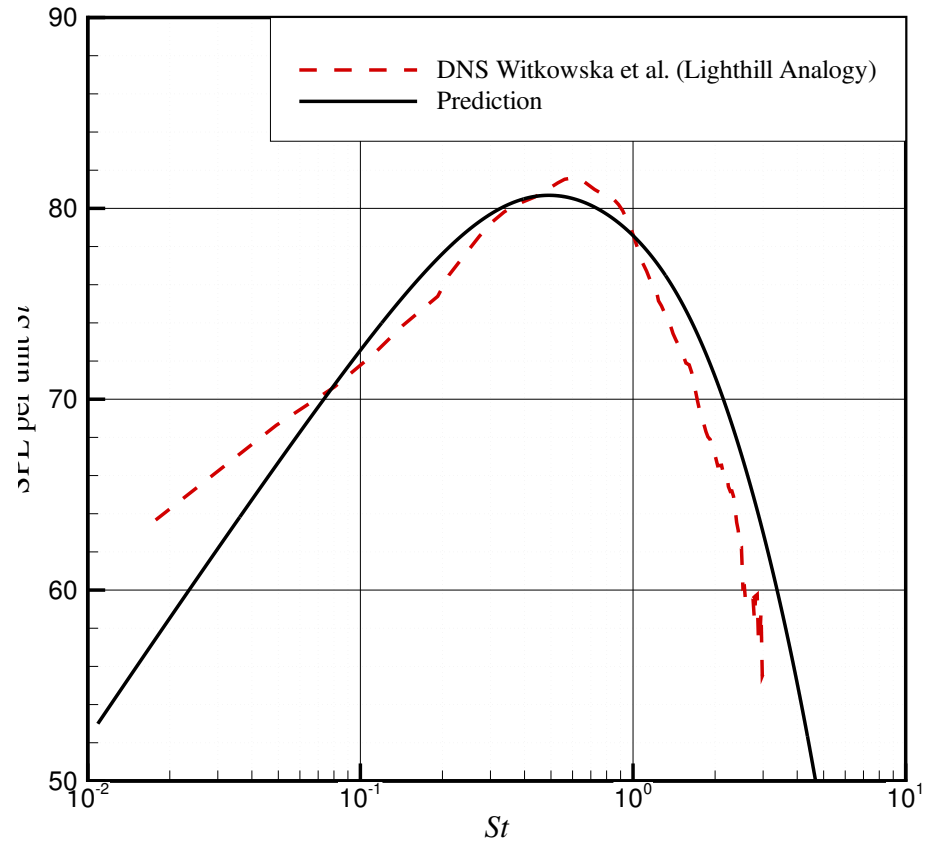


Interaction of Instability Waves



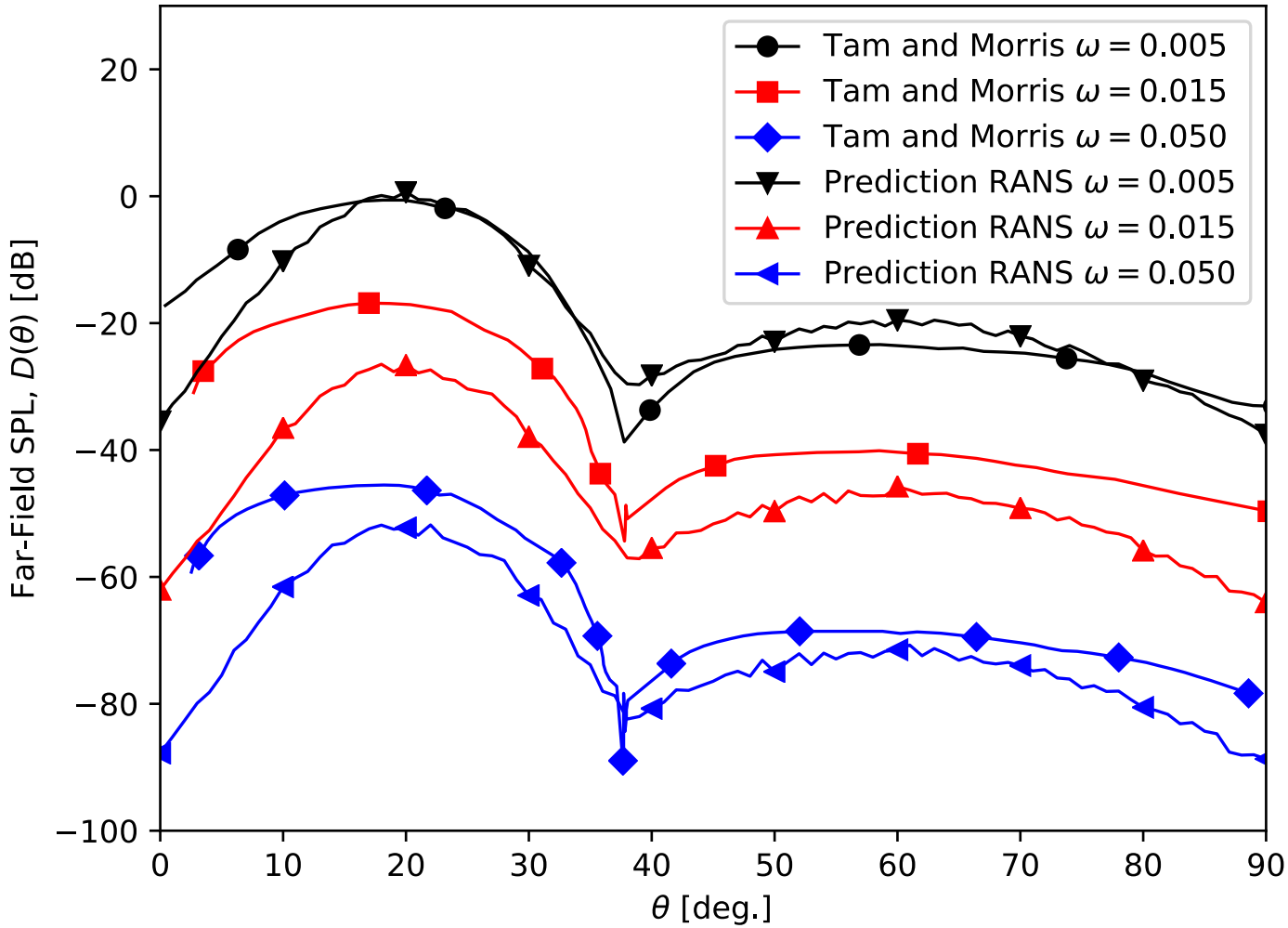
Previous Predictions for Isotropic Turbulence



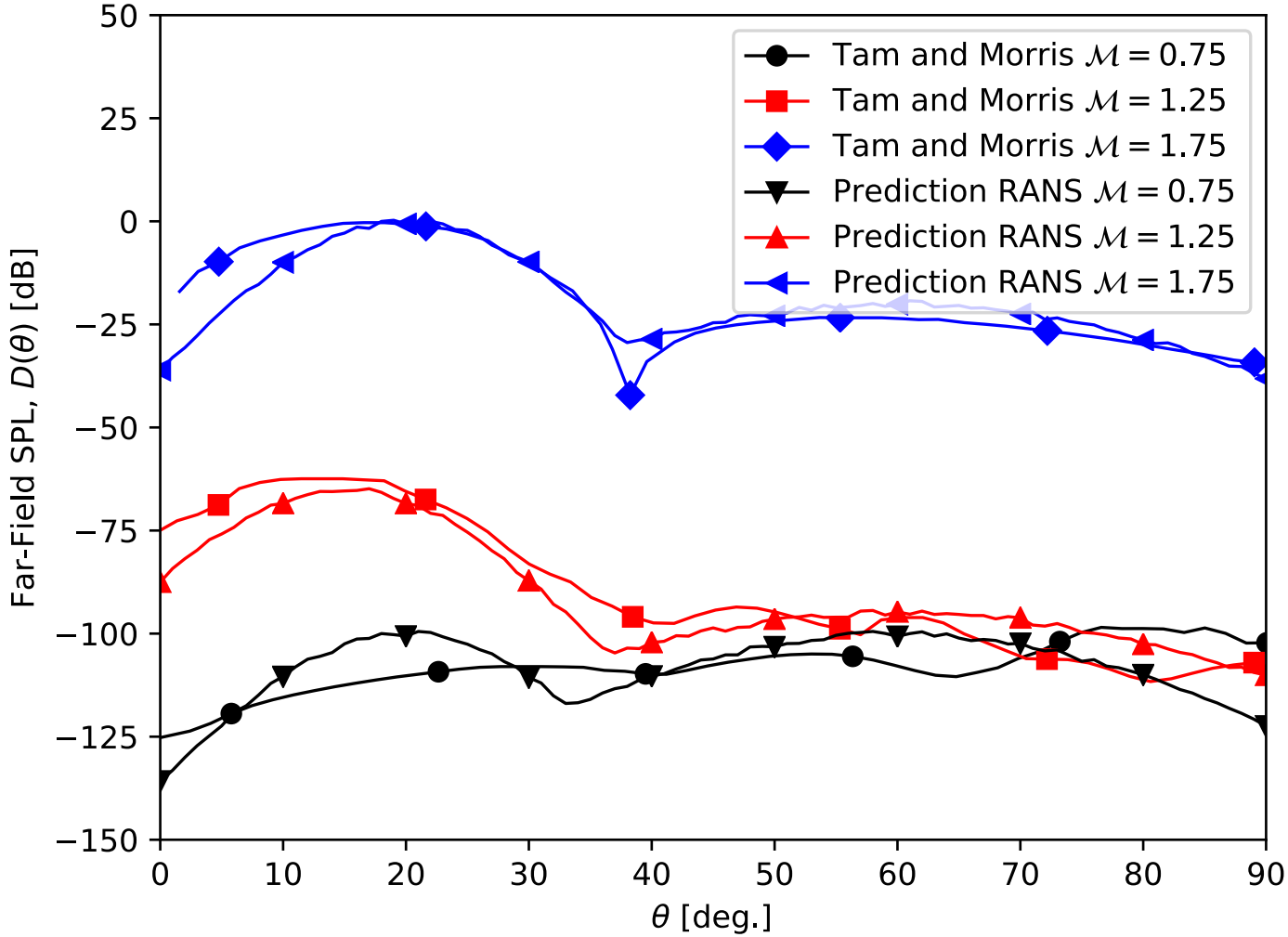


Published this year in AIAAJ by Miller (single author)

Acoustic Prediction at Mach 1.75 RANS Based Sources



Acoustic Prediction
RANS Based Sources



Summary and Future Research

Summary

- Previously made predictions for isotropic turbulence (analytical approach)
- Conducted predictions for large-scale turbulence associated noise for 2D shear layer
- Captures large-scale structure and large-scale structure interactions
- RANS and LES based source models

Future Research

- Extend model to anisotropic-isotropic sources
- Extend model to large-scale structures interacting with shock waves
- Perform simulations of 3D off-design jets and predict noise
- Examine radiated noise, turbulence statistics, as function of nozzle conditions and geometry
- Future Seminars
  - Weiqi Shen – LES and Decomposition
  - Trushant Patel – Aeroacoustic Modeling

Thank You

"My dear, here we must run as fast as we can, just to stay in place. And if you wish to go anywhere you must run twice as fast as that."

Through the Looking Glass (and what Alice found there), Lewis Carroll (Rev. Charles Dodgson)