Extraction of Large-Scale Coherent Structures from Large Eddy Simulation of Supersonic Jets for Shock-Associated Noise Prediction

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Outline

➢ Introduction
➢ Mathematical Methods
➢ Numerical Methods
➢ Results
➢ Summary and Conclusions
Introduction

Motivation and backgrounds

- Failure of modern hearing protections
- Service-related health concerns
- Cost over 1 billion dollars each year
- Future supersonic commercial airliner

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Peak jet noise levels of modern high-performance aircrafts. Figure from NRAC Report 2009.

Supersonic jet noise

Noise components of supersonic jets.

Typical noise spectrum of a supersonic jet.

\[ \phi = 130^\circ \]
Previous studies on BBSAN

- Harper-Bourne and Fisher [1]: discrete shock-vortex interaction
- Tam [2]: stochastic model for BBSAN
- Morris and Miller [3]: RANS based acoustic analogy
- Liu et al. [4]: numerical investigation of BBSAN using LES
- Miller [5]: cross spectral acoustic analogy
- Suzuki [6]: wave-packet like model for BBSAN using LES data

Present approach

Decomposition of N-S equations → BBSAN source terms → Reconstructed flow-field → Source analysis → BBSAN spectra

Time resolved flow-field → Acoustic prediction → Experimental measurement

High-order LES solver → Time resolved flow-field

Combined analysis

validate
Mathematical Methods

Decomposition of N-S equations \[^1\]

\[ q = \overline{q} + q' + \hat{q} + \bar{q}, \quad \bar{q} = \overline{q} + \hat{q} + \bar{q} \]

\[
\frac{\partial q'}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho' u_j + \rho u_j' \right) = \Theta_0
\]

\[
\frac{\partial}{\partial t} \left( \rho' u_i + \rho u_i' \right) + \frac{\partial}{\partial x_j} \left[ \rho' u_i u_j + \rho u_i' u_j + \rho u_j u_i' + p' \delta_{ij} - \tau_{ij}' \right] = \Theta_i
\]

\[
\frac{\partial q'}{\partial t} + \frac{\gamma - 1}{2} \frac{\partial}{\partial t} \left( \rho' u_k u_k + 2 \rho u_k' u_k \right) \\
+ \frac{\gamma - 1}{2} \frac{\partial}{\partial x_j} \left[ \left( \rho' u_j u_k u_k + \rho u_j' u_k u_k + 2 \rho u_j u_k' u_k \right) \right] \\
+ \gamma \frac{\partial}{\partial x_j} \left[ (u_j' p + u_j p') \right] - (\gamma - 1) \frac{\partial}{\partial x_j} \left[ -q'_j + u_i \tau'_{ij} + u_i \tau_{ij} \right] = \Theta_4
\]

Field-variables are decomposed into:

\(\overline{\cdot}\) – time averaged
\(\cdot'\) – acoustic perturbation
\(\hat{\cdot}\) – anisotropic turbulent fluctuation
\(\bar{\cdot}\) – isotropic turbulent fluctuation

Equivalent source terms

\[ \Theta_0 = - \frac{\partial p}{\partial t} - \frac{\partial (\rho u_j)}{\partial x_j} \]

\[ \Theta_i = - \frac{\partial}{\partial t} \left( \rho u_j \right) - \frac{\partial}{\partial x_j} \left( \rho u_j u_j + p \delta_{ij} - \tau_{ij} \right) \]

\[ \Theta_4 = - \frac{\partial p}{\partial t} \frac{\gamma - 1}{2} \frac{\partial (\rho u_k u_k)}{\partial t} - \frac{\partial}{\partial x_j} \left[ \frac{\gamma - 1}{2} \rho u_j u_k u_k + \gamma u_j p \right] - (\gamma - 1) \frac{\partial}{\partial x_j} \left[ q_j - u_i \tau_{ij} \right] \]

Decomposition of N-S equations

Acoustic pressure is calculated by

\[ p'(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{n=0}^{4} g^n_p(x, y; t, \tau) \Theta_n(y, \tau) d\tau dy \]

Where \( g^n_p \) is the vector Green’s function of acoustic pressure.

Power spectral density of acoustic pressure is

\[ S_p(x, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \sum_{m=0}^{4} \sum_{n=0}^{4} G^n_p(x, y; f) G^m_p(x, z; f) \int_{-\infty}^{\infty} R_{mn}(y, z; \Delta\tau) e^{-i2\pi f(\Delta\tau)} d\Delta\tau \right] dy dz \]

\[ R_{mn}(y, z; \Delta\tau) = \int \Theta_m(y, \tau + \Delta\tau) \Theta_n(z, \tau) d\tau \]

where \( G^m_p \) is the Fourier transformed vector Green’s function of acoustic pressure,

\( R_{mn} \) is the two-point cross-correlation of source terms.
BBSAN source term

- **Noise source of BBSAN** \(^{[1]}\)
  \[ \Theta_s = -\gamma \hat{u}_j \frac{\partial \bar{p}}{\partial x_j} \]

- **Simplified to wave equation**
  \[ \frac{1}{c_\infty^2} \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i^2} = \frac{\partial \Theta_s}{c_\infty^2 \partial t} \]

- **BBSAN power spectrum**
  \[ S_p(x, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_p^s(x, y; f) G_p^s*(x, z; f) S_{ss}(y, z, f) dy \, dz \]

Where
  \[ S_{ss}(y, z, f) = \hat{\Theta}_s(y, f) \hat{\Theta}_s^*(z, f) \]

\[ G_p^s(x, y; f) = \frac{\exp(-2\pi if|x - y|/c_\infty)}{2c_\infty^2|x - y|} \]


Proper orthogonal decomposition

- \[ \int S(x, x'; f)\Phi(x', f)dx' = \lambda(f)\Phi(x, f) \] \[ \text{[1]} \]
- Computational procedure by Towne et al. \[ \text{[2]} \]

Preprocess raw data

\[
\begin{bmatrix}
q_1, q_2, q_3, \ldots, q_{N_f}, q_{N_f+1}, \ldots, q_{2N_f-N_0}, \ldots, q_N
\end{bmatrix} \rightarrow \begin{bmatrix}
Q^{(1)}, Q^{(2)}, \ldots, Q^{(N_b)}
\end{bmatrix}
\] \[ \text{(1)} \]

\[
\hat{q}_k^{(l)} = \sum_{j=1}^{N_f} \omega_j q_j^{(l)} e^{-2\pi i (j-1)(k-1)/N_f} \rightarrow \hat{q}_{fk} = \frac{1}{\sqrt{N_f \sum_{j=1}^{N_f} \omega_j^2}} \begin{bmatrix}
\hat{q}_k^{(1)}, \hat{q}_k^{(2)}, \ldots, \hat{q}_k^{(N_b)}
\end{bmatrix}
\]

\[
\frac{1}{\sqrt{N_b}} \sqrt{W} \hat{Q}_{fk} = \begin{bmatrix} \sqrt{W} \Phi_{fk}^* \end{bmatrix} \Sigma_{fk} \Psi_{fk}^H
\]

\[
a^*_fk = \hat{Q}_{fk}^H \Phi_{fk} \rightarrow q^{(l)}(x, k\Delta t) = \sum_{m=1}^{N_f} \sum_{n=1}^{N_b} a^{(l)}_{mn} \Phi_{mn} e^{2\pi i (m-1)(k-1)/N_f}
\]


Numerical Methods

High Fidelity LES solver – HiFiLES \[^1\]

- 2/3D compressible Navier-Stokes solver
  - Energy-stable flux reconstruction scheme
  - Support unstructured hybrid mesh
  - 5 types of elements (tri, quad, tet, pris, hex)
  - Explicit time stepping LSRK45
  - Parallelization through MPI

- Large eddy simulation
  - Smagorinsky model
  - WALE model
  - Similarity models

- Modifications \[^2\]
  - Shock capturing method
  - HLLC Riemann solver
  - Numerical probes


\[^2\] Weiqi Shen, Steven A. E. Miller, "Validation of a High-order Large Eddy Simulation Solver for Acoustic Prediction of Supersonic Jet Flow", Journal of Theoretical and Computational Acoustics, 2020 (Submitted for publication)
Governing equations

- Favre-filtered N-S equation

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_j}{\partial x_j} = 0
\]

\[
\frac{\partial \bar{\rho} \bar{u}_i}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i \bar{u}_j}{\partial x_j} + \bar{\rho} \delta_{ij} - \bar{\tau}_{ij} - \tau_{ij}^{sgs} = 0
\]

\[
\frac{\partial \bar{\rho} \bar{e}_o}{\partial t} + \frac{\partial \bar{\rho} \bar{e}_o \bar{u}_j}{\partial x_j} = 0
\]

Where

\[
\tau_{ij}^{sgs} = 2\mu_{\tau} \left( \bar{S}_{ij} - \frac{1}{3} \bar{S}_{mm} \delta_{ij} \right) - \frac{2}{3} \bar{\rho} k \delta_{ij}
\]

\[
\bar{\tau}_{ij} = 2\mu \left( \bar{S}_{ij} - \frac{1}{3} \bar{S}_{mm} \delta_{ij} \right)
\]

\[
\bar{q}_j = -\gamma \left( \frac{\mu}{Pr} \right) \frac{\partial \bar{\varepsilon}}{\partial x_j}
\]

\[
q_j^{sgs} = -\gamma \left( \frac{\mu_{\tau}}{Pr_{\tau}} \right) \frac{\partial \bar{\varepsilon}}{\partial x_j}
\]

- LES Sub-grid scale model

TKE term taken into pressure

\[
\frac{2}{3} \bar{\rho} k \delta_{ij} \rightarrow \bar{\rho} \delta_{ij}
\]

Wall adapted local eddy-viscosity (WALE) model \[1\]

\[
\mu_{\tau} = \rho \Delta_s^2 \frac{(s_{ij}^d s_{ij}^d)^{\frac{3}{2}}}{(s_{ij}^d s_{ij}^d)^{\frac{5}{2}} + (s_{ij}^d s_{ij}^d)^{\frac{5}{4}}}
\]

Where

\[
s_{ij}^d = \frac{1}{2} \left( \bar{g}_{ij}^2 + \bar{g}_{ji}^2 \right) - \frac{1}{3} \bar{g}_{mm}^2 \delta_{ij}
\]

\[
\bar{g}_{ij}^2 = \frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_j}
\]

\[
\Delta_s = C_w V^{1/3}
\]

\[
C_w = 0.325
\]

FWH acoustic solver

Ffowcs-Williams and Hawkings equation \[1\]

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2} \right) [p' \mathcal{H}(f)] = \frac{\partial}{\partial t} [\rho_\infty U_n \delta(f)] - \frac{\partial}{\partial x_i} [L_i \delta(f)] + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij} \mathcal{H}(f)]
\]

Where

\[U_i = \left(1 - \frac{\rho_*}{\rho_\infty}\right) v_i + \frac{\rho_* u_i}{\rho_\infty}\]

\[L_i = p' n_i + \rho_* u_i (u_n - v_n)\]

Density correction for hot jets by Spalart and Shur \[2\]

\[\rho_* = \rho_\infty + p'/c^2_\infty\]

Solution by Farassat \[3\]

\[p'(x, t) = \frac{1}{4\pi} \int \left[ \frac{\rho_\infty U_n}{r} + \frac{L_r}{c_\infty r} + \frac{L_r}{r^2} \right]_{ret} dS\]

- Integrand evaluated at retarded time
- Notations
  - \(r\) – distance between source and observer
  - \(\Delta\) – derivative with respect to source time
  - \(n\) – surface normal direction component
  - \(r\) – observer direction component

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Results

Simulation setup
- Hot under-expanded supersonic jet
- Converging nozzle (SMC000)
- $M_j = 1.47$, TTR=3.2
- Experimental data from SHJAR [1]
- Conical frustum domain

Boundary conditions
- Nozzle walls: adiabatic non-slip wall
- Outer boundaries: Riemann invariant far-field

Simulation setup

<table>
<thead>
<tr>
<th>$\Delta x/D$ Flow region</th>
<th>$\Delta x/D$ Near field</th>
<th>No. DOFs ($\times 10^6$)</th>
<th>No. cells ($\times 10^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/D = 0$</td>
<td>$x/D = 25$</td>
<td>$x/D = 0$</td>
<td>$x/D = 35$</td>
</tr>
<tr>
<td>0.025</td>
<td>0.35</td>
<td>0.075</td>
<td>0.45</td>
</tr>
</tbody>
</table>

- 2.5M tetrahedra elements
- Polynomial order $\mathcal{P}=3$
- $St_{\text{max}} = \frac{(P+1)C_{\infty}D}{8\Delta x U_j} \leq 1.5$
- $y^+ \sim 280$ on nozzle internal wall
- Isotropic elements in freestream

Parameters of computational grids.

Grid refinement schematic.

Computational grid.
Simulation setup

- FWH surface

FWH surface with x-momentum

<table>
<thead>
<tr>
<th>x-momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0E+02</td>
</tr>
<tr>
<td>6.2E+02</td>
</tr>
<tr>
<td>5.4E+02</td>
</tr>
<tr>
<td>4.6E+02</td>
</tr>
<tr>
<td>3.8E+02</td>
</tr>
<tr>
<td>3.0E+02</td>
</tr>
<tr>
<td>2.2E+02</td>
</tr>
<tr>
<td>1.5E+02</td>
</tr>
<tr>
<td>6.6E+01</td>
</tr>
<tr>
<td>1.4E+01</td>
</tr>
<tr>
<td>3.3E+01</td>
</tr>
</tbody>
</table>

FWH surface with mesh

- Encompass all the noise sources
- Placed within the refinement region
- End cap averaging technique [1]
  - 11 endcaps
  - Reduce spurious noise $0.008 < St < 0.08$

Simulation results

- Instantaneous flow-field

Fluctuating pressure

Density
Simulation results

- **Acoustic validation**

  - Sampling interval: $5.5 \times 10^{-6}$ sec
  - Max accessible Strouhal number: $St_{\text{max}} = 6.5$
  - Azimuthal averaged over 12 stations

**Results**

- Good agreement at upstream and downstream angles
- BBSAN, large-scale mixing noise, and screech well captured
- Overprediction of noise at $\phi = 70^\circ$
  - Insufficient grid resolution
  - Laminar nozzle boundary layer

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FWH surface sampling setup

Simulation results

Data sampling volume

- $x/D \in [0.1,13]$, $y/D = z/D \in [-3,3]$
- $\Delta x = \Delta y = \Delta z = 0.1D$

• Velocity vector ([u v w])
• 512 snapshots per block with 50% overlap
• Low rank behavior around $St=0.21$ (screech)

POD eigenvalues as a function of Strouhal number, normalized by total flow energy.
Leading POD modes

St=0.09

St=0.30 (peak BBSAN frequency at 130°)

St=0.21 (screech frequency)

St=0.50 (peak BBSAN frequency at 90°)
Spectral shape and amplitude of BBSAN well predicted
Fundamental screech captured in the BBSAN spectra
BBSAN spectra calculated with different number of POD modes

- Fewer POD modes needed to preserve the spectral shape of the primary lobe at larger observation angles where the width of the lobe are smaller.
- Peak frequencies predicted with only the leading POD mode
BBSAN source distribution
\( \Phi = 90^\circ, St = 0.5 \)

\[
I_p(x, y; f) = |G_p^s(x, y; f)| \int_{-\infty}^{\infty} G_p^{*}(x, z; f)S_{ss}(y, z, f) \, dz
\]

- Source locations agree with previous measurements by Podboy \(^1\)

Normalized cross-correlation of axial fluctuating velocity at source locations

\[ R_{uu}(\mathbf{y}, \mathbf{z}; \tau) = \int u(\mathbf{y}, t + \tau)u(\mathbf{z}, t)dt \]

- Correlation length scale and time scale increase as flow propagates downstream
- Increasement of convective velocity in the downstream direction
Normalized cross-correlation of axial fluctuating velocity at source locations

\[ R_{uu} \text{ RANS } x/D=3.0 \]

- Ribner’s model \[^1\] : \[ R = \exp(-|\tau|/\tau_s) \exp(-\left(\eta - u_c \tau\right)^2/l^2) \]
- RANS predicts smaller correlation length scales and larger correlation time scales

Source term correlations at source locations

\[ R_{SS}(y, z; \tau) = \int \Theta_S(y, t + \tau) \Theta_S(z, t) dt \]

- \( R_{SS} \) of LES at \( x/D = 3.0 \)
  - Strong positive correlation paired with negative correlation due to the shock wave shear layer interaction.

- \( R_{SS} \) of RANS at \( x/D = 3.0 \)
Source term correlations at source locations

- $R_{SS}$ of LES at $x/D=4.5$
  - Correlation with the neighboring shock wave are observed in LES due to growth in time scales

- $R_{SS}$ of RANS at $x/D=4.5$
Summary and Conclusion

- Simulated and validated an off-design heated supersonic jet
- Used POD to extract large-scale structures from the flow-field
- Validated BBSAN source term by comparing BBSAN spectra with total spectra
- Sources of BBSAN are located on the shock waves and at the end of the potential core
- Stronger source correlation between shocks waves as the flow proceeds downstream

Future Work

- Investigate the overprediction of BBSAN at low frequencies.
Thank you. Questions?