Numerical Evaluation of Noise Sources and Statistics from High-Speed Two-Phase Jet Flow

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Outline

- Introduction
- Computational Approach
- Results and Discussions
- Summary and Conclusions
- Future Work
Introduction
Rocket Exhaust Noise


Delta IV rocket lifts off from Space Launch Complex 37 at Cape Canaveral Air Force Station in Florida for an unpiloted test flight of NASA's Orion spacecraft. Image is obtained online from nasa.gov.

Figures and data are from Jayanta Panda from NASA Ames Research Center.
Acoustic Analogies

- Aerodynamic noise theory in which the equations of motion for a compressible fluid are rearranged in a way that separates linear acoustic propagation effects[1].

- Lighthill

- Curle and Ffowcs-Williams and Hawking (FW-H)

- Phillip, Lilley, and Goldstein

- Crighton and Ffowcs-Williams for Two-phase Flow[2]

\[
\left( \frac{\partial^2}{\partial t^2} - c_\infty^2 \nabla^2 \right) (\rho - \rho_\infty) = \frac{\partial Q}{\partial t} - \frac{\partial G_i}{\partial x_i} + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

**Effect of volume fraction**

Monopole: \( Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha) \)

**Effect of drag force**

Dipole: \( G_i = F_i + \frac{\partial}{\partial t} \alpha \rho v_i \)


Selected Previous Investigations

- NASA Marshall
  - Ares I Scale Model Acoustic Test (ASMAT)\textsuperscript{[1]}
- Loci/CHEM code
- NASA AMES
  - Launch, Ascent, and Vehicle Aerodynamics (LAVA) code\textsuperscript{[2]}

Selected Previous Investigations

- **University of Michigan**
  Sound Generated in Compressible Shear Layers\(^1\)

- **NASA**
  Near-field noise of solid-propellant rocket\(^2\)

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Computational Approach
Computational Method

Implicit LES

CFD solver
Particle solver

2-way coupling

Aerodynamic

Time-dependent flow-field
Surface

Aeroacoustics

Acoustic solver (FW-H)
Source analysis

Far-field noise

Numerical Simulation

CFD Solver
- RocfluidMP CFD Solver
- Unstructured
- Finite Volume
- Eulerian-Lagrangian
- 2-Way Coupling
- Implicit LES
- AUSM+
- 3rd order Runge-Kutta

Acoustics Solver
- Developed in-house
- Ffowcs-Williams and Hawking’s (FW-H)
- Far-field acoustics

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Governing Equations

Navier-Stokes Equation

\[
\frac{\partial}{\partial t} \int_{\Omega} (\mathbf{1} - \mathbf{\alpha}) \mathbf{W} d\Omega + \int_{\partial \Omega} (\mathbf{1} - \mathbf{\alpha}) \mathbf{F}_c dS = \int_{\partial \Omega} (\mathbf{1} - \mathbf{\alpha}) \mathbf{F}_v dS + \int_{\Omega} \mathbf{Q} d\Omega
\]

where

\[
\mathbf{W} = [\rho, \rho u, \rho v, \rho w, \rho E]^T
\]

\[
\mathbf{F}_c = [\rho V_n, \rho u V_n + n_x p, \rho v V_n + n_y p, \rho w V_n + n_z p, \rho H V_n]^T
\]

\[
\mathbf{F}_v = \begin{bmatrix}
0 \\
\begin{bmatrix}
\tau_{xx} + \tau_{xy} + \tau_{xz} \\
\tau_{yx} + \tau_{yy} + \tau_{yz} \\
\tau_{zx} + \tau_{zy} + \tau_{zz} \\
\Theta_x + \Theta_y + \Theta_z
\end{bmatrix}
\end{bmatrix}
\]

\[
\Theta_x = u \tau_{xx} + v \tau_{xy} + w \tau_{xz} + k \frac{\partial T}{\partial x}
\]

\[
, \quad \Theta_y = u \tau_{yx} + v \tau_{yy} + w \tau_{yz} + k \frac{\partial T}{\partial y}
\]

\[
, \quad \Theta_z = u \tau_{zx} + v \tau_{zy} + w \tau_{zz} + k \frac{\partial T}{\partial z}
\]
Governing Equations

Phase-Coupling Terms
\[ Q = [0, f_{p,x}, f_{p,y}, f_{p,z}, E_p]^T \]
\[ f_p = -\sum m_p \frac{(V - V_p)}{\tau_u} \]
\[ E_p = \sum [f_p \cdot (V_p - V)] \]
\[ - \frac{m_p C_{p,p} (T - T_p)}{\tau_\theta} \]

Point Particles

- Evolution Equations
  \[ \frac{d}{dt} x_p = V_p, \quad \frac{d}{dt} V_p = \frac{V - V_p}{\tau_u}, \]
  \[ \frac{d}{dt} T_p = \frac{T - T_p}{\tau_\theta} \]

- Time Scales
  \[ \tau_u = \frac{\rho_p d_p^2}{18 \mu f_u(Re)}, \quad \tau_\theta = \frac{c_{p,p} \rho_p d_p^2}{12 k f_\theta(Re)} \]

- Correlations
  Naumann and Schiller(1935):
  \[ f_u(Re) = 1 + 0.15 Re^{0.687} \]
  Ranz and Marshall(1952):
  \[ f_\theta(Re) = 1 + 0.3 Re^{\frac{1}{2}} Pr^{\frac{1}{3}} \]
Ffowcs-Williams Hawking Equation

Inhomogeneous Wave Equation Derived from Navier-Stokes Equation

\[ \Box^2 [p'H(f)] = \frac{\partial}{\partial t} [\rho_0 v_n \delta(f)] + \frac{\partial}{\partial t} [\rho (u_n - v_n) \delta(f)] \]
\[ - \frac{\partial}{\partial x_i} [\rho u_i (u_n - v_n) \delta(f)] + \frac{\partial}{\partial x_i} [L_i \delta(f)] \]
\[ + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij} H(f)] \]

Where

\[ \Box^2 = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2} \]
\[ L_i = -[(p - p_0) \delta_{ij} - \sigma_{ij}] \hat{n}_j \]
\[ T_{ij} = \rho u_i u_j - \sigma_{ij} + [(p - p_0) - c_0^2 \rho'] \delta_{ij} \]
Ffowcs-Williams Hawking Equation

Farrasat’s Formulation\cite{1}

\[
4\pi p'(x, t) = \int \left[ \frac{\rho_0 v_n}{r(1 - M_r)^2} + \frac{\rho_0 v_n \left( r\dot{M}_r + c(M_r - M^2) \right)}{r^2 (1 - M_r)^3} + \frac{l_r}{cr(1 - M_r)^2} + \frac{l_r - l_i \cdot M_i}{r(1 - M_r)^2} + \frac{l_r \left( r\dot{M}_r + c(M_r - M^2) \right)}{cr^2 (1 - M_r)^3} + \left( \frac{\dot{\rho}(u_n - v_n) + \rho(u_n - v_n)}{r(1 - M_r)^2} \right) (1 + M_r) + \frac{\rho(u_n - v_n)}{r(1 - M_r)^2} \cdot \dot{M}_r + \frac{\rho(u_n - v_n)(u_r - u_i M_i)}{r^2 (1 - M_r)^2} + \frac{\rho(u_n - v_n)(1 + M_r)}{r^2 (1 - M_r)^3} \left( r\dot{M}_r + c(M_r - M^2) \right) \right]_{ret} dS
\]

\[ r = |x - y| \]

Subscripts:

\( n \) = surface normal direction

\( r \) = radiation direction

Results and Discussions
Geometry and Computational Grid

- SMC016 Nozzle (SHJAR)[1]
- $D = 0.0508 \, m, M_j = 1.5$
- Smooth Nozzle Wall
- $50D \times 30D$
- 2.1M hexahedra cells
- O-H Structured-like Grid
- Minimum cell $\sim 10^{-4}[m] \sim 1\%D$

Flow Conditions

- Reproduction of SHJAR dataset
- On-design
- Cold jet
- Particle mass flow rate ~ 10% gas mass flow rate

<table>
<thead>
<tr>
<th>Case</th>
<th>NPR</th>
<th>TTR</th>
<th>Fully expanded Mach number, $M_j$</th>
<th>Particle Mass Flux ($kg/m^2s$)</th>
<th>Particle Diameter ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.67</td>
<td>1</td>
<td>1.5</td>
<td>0</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>3.67</td>
<td>1</td>
<td>1.5</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>
Mean Streamwise Velocity

a) Mean streamwise velocity

b) Radial profile of mean velocity

c) Pressure contour


Comparison of the Mean Streamwise Velocity of Gas and Particles

a) Mean streamwise velocities of gas and particles on the centerline

b) Mean velocity profiles of gas and particles in the radial direction
Comparison of Far-field SPL Spectra

- Far-field SPL spectra at downstream direction
- $\theta = 160^\circ, \frac{R}{D} = 100$
- Large-scale mixing noise
- Mach wave radiation
- Attenuation at St below 1
Statistics of $T_{11}$ of Lighthill’s Stress Tensor on the Nozzle Lipline

Sound sources are greatly altered in the particle-laden jet

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Two-Point Cross-Correlation of $T_{11}$ of Lighthill’s Stress Tensor on the Nozzle Lipline

- Highly correlated in small region
- Semi-periodic in single-phase jet
- De-correlated in two-phase jet
- Length scale difference
- Different source compactness

Zero time-lagged two-point cross-correlation.
Reference point location: $X = D, Y = D/2, Z = 0$. 

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Convection of $T_{11}$ of Lighthill’s Stress Tensor

- Convective velocity is reduced in the particle-laden jet.

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Phase Velocity of $T_{11}$ of Lighthill’s Stress Tensor

- Nozzle lipline
- Frequency-wavenumber space
- Phase-velocity $\omega/k$ is lower than convective velocity
- Reduced convective velocity in the downstream direction
- Reduced phase-velocity in the particle-laden jet

![Graphs showing phase velocity in gas jet and particle-laden jet](image)
Monopole and Dipole Sound Source

Crighton and Ffowcs Williams’ acoustic analogy\cite{1}

- **Monopole:**
  \[ Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha) \]

- **Dipole:**
  \[ G_i = F_i + \frac{\partial}{\partial t} \alpha \rho v_i \]

- Monopole source is not zero due to slip effect of the particles

Time-lagged Two-point Correlation of Monopole and Dipole Sound Source

a) Monopole source on the centerline

Reference point location: $X = 10D, Y = 0, Z = 0$.

b) Dipole source on the centerline

Reference point location: $X = 10D, Y = 0, Z = 0$.

c) Monopole source on the lipline

Reference point location: $X = 10D, Y = D/2, Z = 0$.

d) Dipole source on the lipline

Reference point location: $X = 10D, Y = D/2, Z = 0$.
Summary and Conclusion
Summary and Conclusion

• Summary
  • FWH method based on implicit LES two-phase gas-particle simulation
  • Comparison of far-field sound spectra of gas and particle-laden jets
  • Source statistics based on the C-FW acoustic analogy

• Important findings
  • Sound spectrum is greatly altered in particle-laden jet
  • The location, convection and phase velocity of Lighthill’s tensor are altered.
  • Monopole and dipole sources arise in two-phase flows.
  • The strength of monopole and dipole sources are significantly smaller than the quadrupole source.
Future Work

• Develop a volumetric integration method for each sources in particle-laden jet
• Evaluate the spectral component of sound generated by the monopole and dipole sources.
• Study the effects of particle size and mass loading
Thank You