Probability Distributions of Sonic Boom Noise Metrics in Isotropic Turbulent Fields

Alexander N. Carr,[1] Joel B. Lonzaga,[2] and Steven A. E. Miller[1]

[1]Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, Fl

[2]Structural Acoustics Branch, NASA Langley Research Center, Hampton, VA, United States

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Motivation

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- **Goal**: Understand how turbulence alters sonic boom metrics
- Reasons:
  - Knowledge of metrics at ground level is required to develop noise standards, plan community response surveys, etc.
  - Turbulence in the atmospheric boundary layer (ABL) randomly alters sonic boom signals
- **Process**: Perform numerical simulations of sonic boom through turbulence, examine probability distributions and mean values of sonic boom metrics
- Reasons:
  - Computational approaches provide a cheap alternative to flight test measurements or laboratory experiments
  - Parameters of simulated turbulent fields can be controlled

Background

- Sonic boom metrics:
  - Stevens' Mark VII Perceived level [1] (PL)
  - Weighted Sound Exposure Levels (A-E weighted SEL)
  - Indoor Sonic Boom Annoyance Predictor [2] (ISBAP)
- Flight test measurements:
  - Bradley et al. [3] (SonicBAT): Average PL and ISBAP decrease with propagation distance
- Laboratory scale experiments:
  - Lipkens and Blackstock [4]: Rise time increases as N-wave propagates through turbulence
- Computational simulations:
  - Stout [5]: Average PL and ISBAP decrease in propagation direction, supported by SonicBAT measurements
- Additional background: Averiyanov et al. [6], Yuldashev et al. [7], Luquet [8]

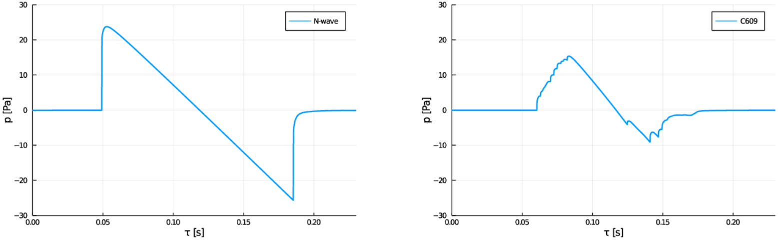
Approach

1. Simulate isotropic velocity fields on structured grid
2. Numerically solve FLHOWARD equation [8] (Eqn. 1) with Strang split step [9]
3. Each physical effect is computed with different numerical method (for more info see [8, 5, 10])
4. Extract sonic boom signals at 342 locations along the x_1 direction, and 25 locations in the transverse directions
5. Compute sonic boom metrics at each microphone location for each simulation
6. Compute average and standard deviations at each x_1 location across 500 simulations (12,500 microphones)

$$\frac{\partial^2 \tilde{p}}{\partial x_1 \partial \tau} = \frac{\bar{c}}{2} \frac{\partial^2 \tilde{p}}{\partial x_i \partial x_i} - \frac{u_1'}{\bar{c}} \frac{\partial^2 \tilde{p}}{\partial x_1 \partial \tau} + \frac{u_1'}{\bar{c}^2} \frac{\partial^2 \tilde{p}}{\partial \tau^2} + \frac{u_2'}{\bar{c}} \frac{\partial^2 \tilde{p}}{\partial x_2 \partial \tau} + \frac{u_3'}{\bar{c}} \frac{\partial^2 \tilde{p}}{\partial x_3 \partial \tau} + \frac{\delta}{2\bar{c}^3} \frac{\partial^3 \tilde{p}}{\partial t^3} + \frac{\beta}{2\overline{\rho c}^3} \frac{\partial^2 \tilde{p}^2}{\partial t^2}. \quad (1)$$

Initial Waveforms

- Two waveforms are considered:
  - N-wave
  - C609 waveform of NASA X-59 QueSST Aircraft
  - Sampled at 12.8 kHz
  - Initially propagated from aircraft to 5000 ft (7000 ft for C609) above the ground in PCBoom
- Spatial domain
  - 0 m $\leq x_1 \leq$ 2046 m, ($\Delta x_1 = 2$ m)
  - -510 m $\leq x_{2,3} \leq$ 510 m, ($\Delta x_{2,3} = 4$ m)



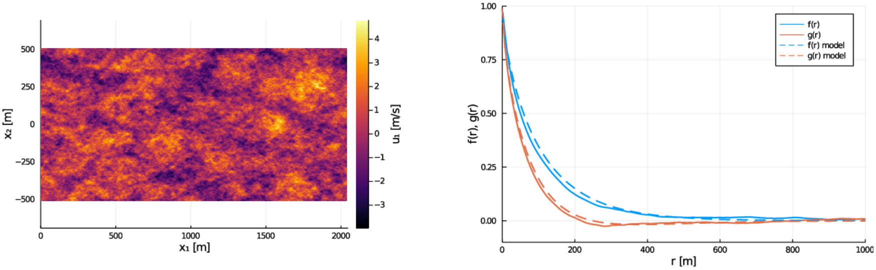
(a) N-wave

(b) C609

Figure 1: Initial waveforms.

Turbulent Field

- Isotropic turbulent field simulated with method of Frehlich [11]
- Temperature fluctuations are not considered
- Longitudinal integral scale L_f and rms velocity σ_u are prescribed for each propagation case considered
- N-wave and C609 signals are propagated through 500 randomly generated fields for each case



(a) 2D slice of u_1 velocity field

(b) Longitudinal and Lateral correlations

Figure 2: a) Turbulent field generated for $\sigma_u = 1.0$ m/s and $L_f = 100$ m. b) Comparison of average longitudinal and lateral correlations with correlation functions associated with the von Kármán spectral model.

Parameter Space

- According to Bradley et al. [3], turbulence intensity levels in the ABL can be categorized by the rms velocity

Table 1: Categories of turbulence intensity based on rms velocity

Turbulence intensity	σ_u m/s
Low	0.31
Medium	1.10
High	1.89

- Based on these categories and other experiments [4, 6]:
  - RMS velocity: 0.2 m/s $\leq \sigma_u \leq$ 3.0 m/s
  - Integral scale: 100 m $\leq L_f \leq$ 200 m

Pre-Shock Noise

- Noise before the initial shock formed in the medium to high intensity propagation cases
- Effect of pre-shock noise was examined on a few waveforms
- Metrics of original waveforms were computed and compared to the same waveform with pre-shock noise eliminated
- Increase in PL and ISBAP. ASEL and CSEL were not altered by more than ± 0.6 dB

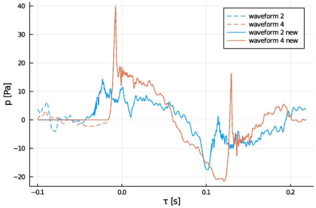
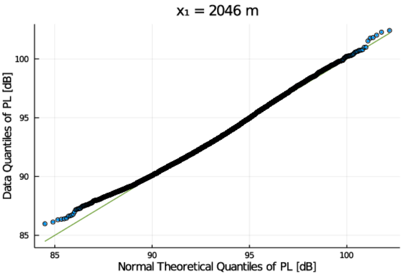


Figure 3: Two waveforms sampled from test simulations with $\sigma_u = 1.8$ m/s and $L_f = 100$ m. The pre-shock noise that originally occured in the waveforms is shown as dashed lines.

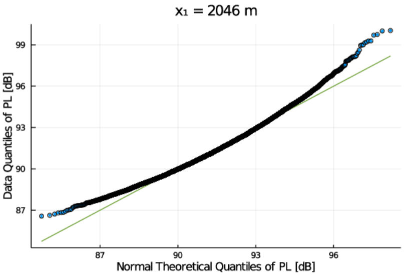
Waveform	ΔPL	ΔISBAP
1	2.2×10^{-5}	-7.2×10^{-4}
2	0.96	0.72
3	1.64	1.58
4	0.09	0.06
5	1.37	1.32

Distributions of Sonic Boom Metrics I

- PL closely follows a normal distribution for lower turbulence levels
- Distribution is skewed to the right at higher turbulence levels
- Figure 4 shows distribution of PL for N-wave



(a) $\sigma_u = 0.2$ m/s



(b) $\sigma_u = 1.8$ m/s

Figure 4: Quantile-Quantile plots of PLdB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

Distributions of Sonic Boom Metrics II

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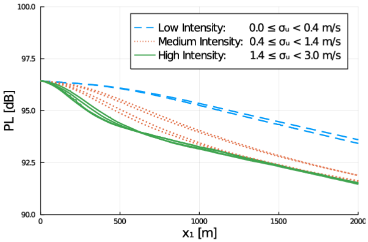
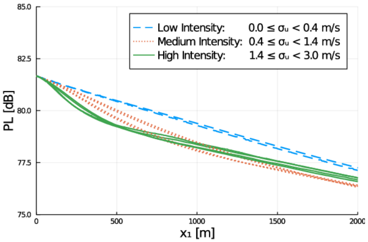
- N-wave:
  - Similar trend occurs for all metrics, except for ASEL and DSEL
  - ASEL is normal for low turbulence levels, skewed to the right for medium turbulence levels, and returns to normal for high turbulence levels
  - DSEL results show skewness at lower turbulence intensities
- C609 waveform:
  - All metrics follow same trend as PL, except ASEL, CSEL and DSEL
  - ASEL follows same trend as N-wave
  - CSEL remains normal for all turbulence levels
  - DSEL is normal at medium turbulence intensities, skewed to the right at low and high intensities

Expected Values I

- Examine average PL as a function of propagation distance and turbulence intensity
- Mean PL computed from several cases conducted at low, medium, and high turbulence intensities
- Nonlinear relationship with x_1 when $x_1 \leq 500$ m
- Beyond 1000 meters, decrease in PL is approximately linear



(a) N-wave



(b) Low Boom waveform

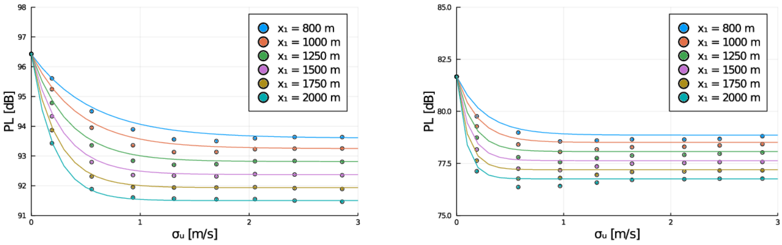
Figure 5: Average value of PL dB as a function of propagation distance for both waveforms.

Expected Values II

- Model mean values beyond $x_1 = 1000$ m
- First, examine decrease with respect to turbulence rms velocity
- Solid lines represent exponential decay model with fitted parameters

$$\mathrm{PL}(\sigma_u) = C_1 \left[C_3 + \exp(-C_2\sigma_u)\right] \quad (2)$$

- Exponential decay model does not capture additional dip seen in C609 waveform results and some N-wave results



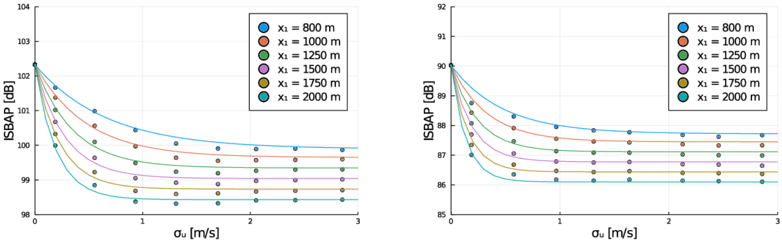
(a) N-wave

(b) C609 waveform

Figure 6: Average value of PLdB as a function of rms velocity for two waveforms.

Expected Values III

- ISBAP shows similar trend to PL
- Mismatch in the range $0.6 \leq \sigma_u \leq 1.8$ m/s for N-wave
- Mismatch for $0.2 \leq \sigma_u \leq 0.6$ m/s for C609 waveform
- Additional metrics (ASEL and CSEL) are shown in the extra slides



(a) N-wave **(b)** C609 waveform

Figure 7: Average ISBAP as a function of σ_u for a) N-wave and b) C609, compared to exponential decay model.

Summary and Conclusions

- Computational results suggest the following:
  - Sonic boom metrics are normally distributed at low turbulence intensities
  - As σ_u increases, distributions become skewed to the right
  - Average values of PL and ISBAP are approximately constant with respect to σ_u when $\sigma_u > 2$ m/s
  - The decrease in average PL and ISBAP with respect to σ_u can be approximated by an exponential decay curve
- Future work:
  - Resolve issues with pre-shock noise
  - Investigate changes to sonic boom metrics when integral scale is increased
  - Present data for standard deviations and 95% confidence intervals in future publication
  - Propagation through inhomogeneous turbulence

Thank you

References I

[1] S. S. Stevens, “Perceived level of noise by mark VII and decibels (e),” *The Journal of the Acoustical Society of America*, vol. 51, pp. 575–601, Feb. 1972.

[2] J. Rathsam, A. Loubeau, and J. Klos, “A study in a new test facility on indoor annoyance caused by sonic booms,” Tech. Rep. NASA/TM–2012-217332, National Aeronautics and Space Administration, Langley Research Center, 2012.

[3] K. A. Bradley, C. M. Hobbs, C. B. Wilmer, V. W. Sparrow, T. A. Stout, J. M. Morgenstern, K. H. Underwood, D. J. Maglieri, R. A. Cowart, M. T. Collmar, and et al., “Sonic booms in atmospheric turbulence (sonicbat): The influence of turbulence on shaped sonic booms,” *NASA Technical Report*, 2020.

[4] B. Lipkens and D. T. Blackstock, “Model experiment to study sonic boom propagation through turbulence. part i: General results,” *The Journal of the Acoustical Society of America*, vol. 103, pp. 148–158, Jan. 1998.

[5] T. Stout, *Simulation of N-wave and shaped supersonic signature turbulent variations*. PhD thesis, Pennsylvania State University, Dec. 2018.

[6] M. Averiyanov, P. Blanc-Benon, R. O. Cleveland, and V. Khokhlova, “Nonlinear and diffraction effects in propagation of N-waves in randomly inhomogeneous moving media,” *The Journal of the Acoustical Society of America*, vol. 129, pp. 1760–1772, Apr. 2011.

[7] P. V. Yuldashev, S. Ollivier, M. M. Karzova, V. A. Khokhlova, and P. Blanc-Benon, “Statistics of peak overpressure and shock steepness for linear and nonlinear n-wave propagation in a kinematic turbulence,” *The Journal of the Acoustical Society of America*, vol. 142, pp. 3402–3415, Dec. 2017.

[8] D. Luquet, *3D simulation of acoustical shock waves propagation through a turbulent atmosphere. Application to sonic boom*. Theses, Université Pierre et Marie Curie - Paris VI, Jan. 2016.

[9] G. Strang, “On the construction and comparison of difference schemes,” *SIAM Journal on Numerical Analysis*, vol. 5, pp. 506–517, Sept. 1968.

[10] A. N. Carr, J. B. Lonzaga, and S. A. Miller, “Modelling sonic boom propagation through planetary boundary layer turbulence near the lateral extent of the carpet,” *The Journal of the Acoustical Society of America*, vol. 148, pp. 2739–2739, Oct. 2020.

[11] R. Frehlich, L. Cornman, and R. Sharman, “Simulation of three-dimensional turbulent velocity fields,” *Journal of Applied Meteorology*, vol. 40, pp. 246–258, Feb. 2001.

Cases I

- These are all of the cases conducted for both N-wave and C609

Table 2: Test Cases for Propagation through Isotropic Turbulence (Suite 1)

Test Cases	σ m/s	L_f m	d_1	$d_{2,3}$	Δx_1	$\Delta x_{2,3}$	f_s kHz
Case 1	0.0	0.0	2046 m	1020 m	2 m	4 m	12.8
Case 2	0.2	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 3	0.6	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 4	1.0	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 5	1.4	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 6	1.8	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 7	2.2	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 8	2.6	100.0	2046 m	1020 m	2 m	4 m	12.8
Case 9	3.0	100.0	2046 m	1020 m	2 m	4 m	12.8

Cases II

Table 3: Test Cases for Propagation through Isotropic Turbulence (Suite 3)

Test Cases	σ m/s	L_f m	d_1	$d_{2,3}$	Δx_1	$\Delta x_{2,3}$	f_s kHz
Case 18	0.2	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 19	0.6	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 20	1.0	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 21	1.4	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 22	1.8	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 23	2.2	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 24	2.6	150.0	2046 m	1020 m	2 m	4 m	12.8
Case 25	3.0	150.0	2046 m	1020 m	2 m	4 m	12.8

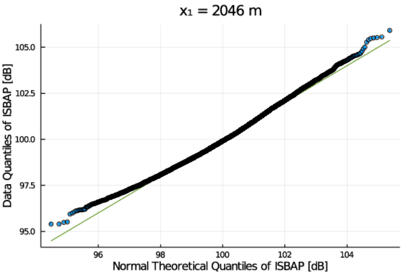
Cases III

Table 4: Test Cases for Propagation through Isotropic Turbulence (Suite 5)

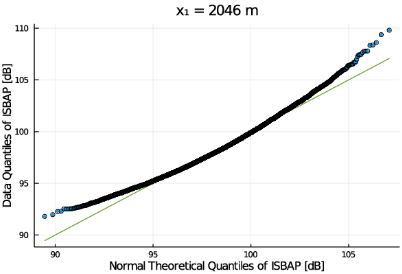
Test Cases	σ m/s	L_f m	d_1	$d_{2,3}$	Δx_1	$\Delta x_{2,3}$	f_s kHz
Case 34	0.2	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 35	0.6	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 36	1.0	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 37	1.4	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 38	1.8	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 39	2.2	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 40	2.6	200.0	2046 m	1020 m	2 m	4 m	12.8
Case 41	3.0	200.0	2046 m	1020 m	2 m	4 m	12.8

ISBAP N-wave

- Similar trend to PL
- ISBAP is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s

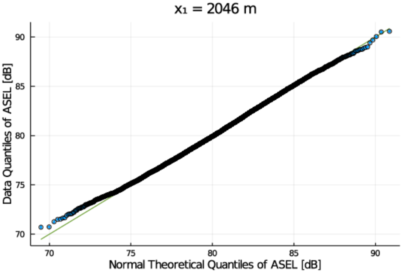


(b) $\sigma_u = 1.8$ m/s

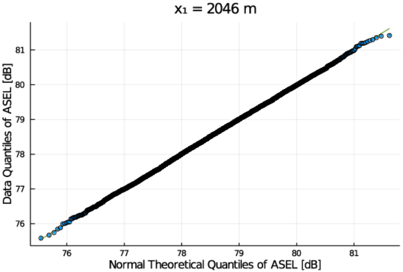
Figure 8: Quantile-Quantile plots of ISBAP dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

ASEL N-wave

- ASEL differs from PL because it follows a normal distribution at high turbulence levels
- At $\sigma_u = 3.0$ m/s ASEL is much closer to the theoretical normal distribution than other metrics



(a) $\sigma_u = 0.2$ m/s

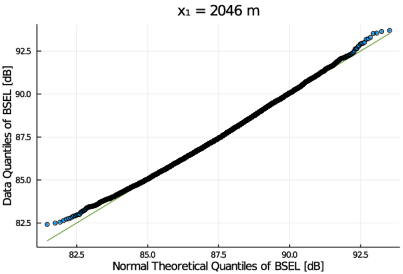


(b) $\sigma_u = 3.0$ m/s

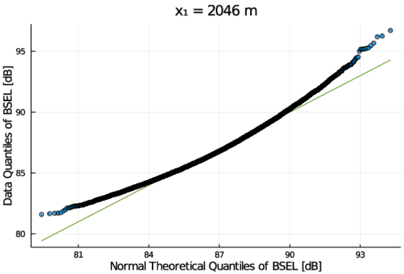
Figure 9: Quantile-Quantile plots of ASEL dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

BSEL N-wave

- Similar trend to PL
- BSEL is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s

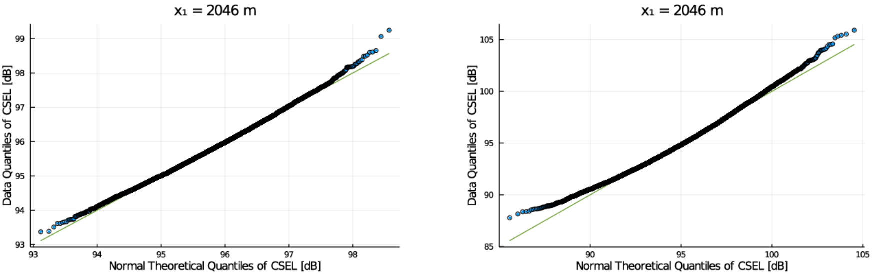


(b) $\sigma_u = 1.8$ m/s

Figure 10: Quantile-Quantile plots of BSEL dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

CSEL N-wave

- Similar trend to PL
- CSEL is skewed to the right at higher turbulence levels



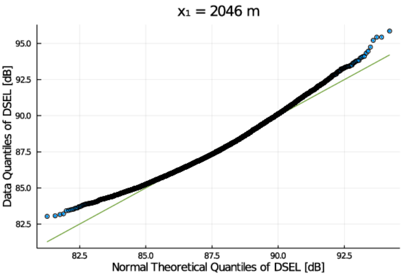
(a) $\sigma_u = 0.2$ m/s

(b) $\sigma_u = 1.8$ m/s

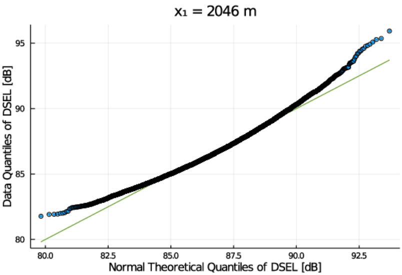
Figure 11: Quantile-Quantile plots of CSEL dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

DSEL N-wave

- DSEL is skewed to the right at all turbulence levels



(a) $\sigma_u = 0.2$ m/s

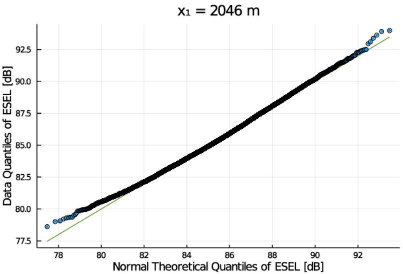


(b) $\sigma_u = 1.8$ m/s

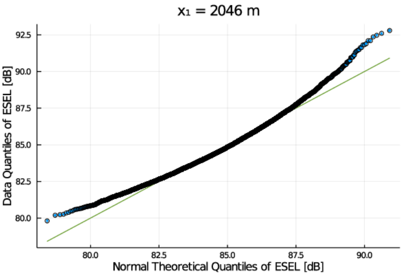
Figure 12: Quantile-Quantile plots of DSEL dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

ESEL N-wave

- Similar trend to PL
- ESEL is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s

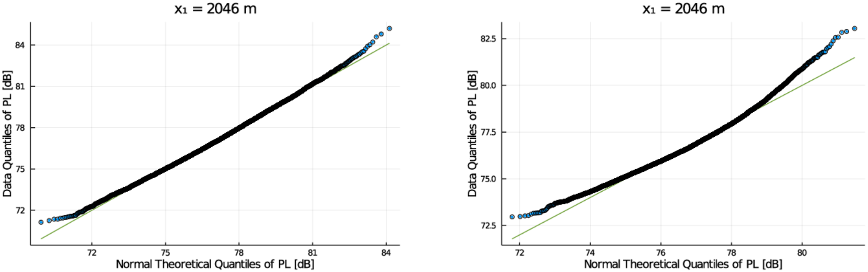


(b) $\sigma_u = 1.8$ m/s

Figure 13: Quantile-Quantile plots of ESEL dB for the N-wave at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

PL C609 Waveform

- Similar trend of PL for the N-wave
- PL is skewed to the right at higher turbulence levels
- Figure 14 for C609 waveform at two turbulence intensities



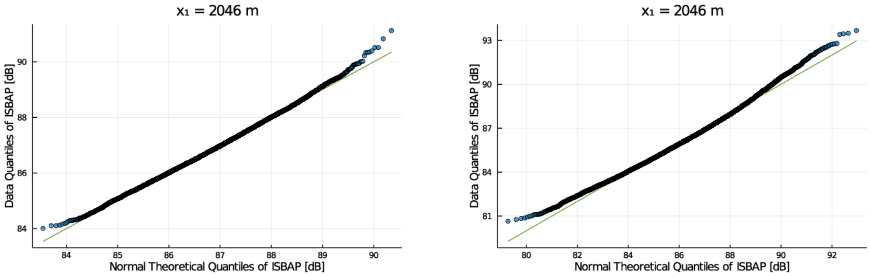
(a) $\sigma_u = 0.2$ m/s

(b) $\sigma_u = 1.8$ m/s

Figure 14: Quantile-Quantile plots of PL dB for the C609 waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

ISBAP C609 Waveform

- Similar trend to PL
- ISBAP is skewed to the right at higher turbulence levels



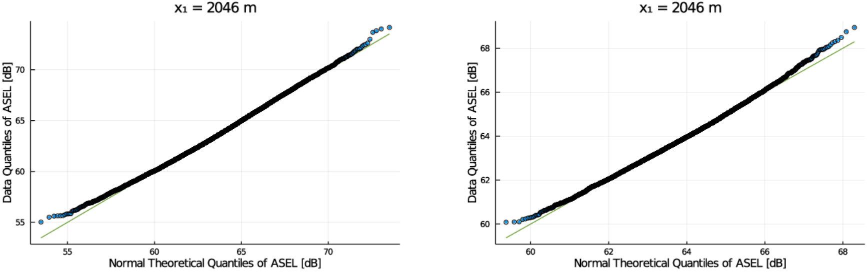
(a) $\sigma_u = 0.2$ m/s

(b) $\sigma_u = 1.8$ m/s

Figure 15: Quantile-Quantile plots of ISBAP dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

ASEL C609 Waveform

- ASEL differs from PL because it follows a normal distribution at high turbulence levels
- At $\sigma_u = 1.8$ m/s ASEL only has a slight skewness, and is much closer to the theoretical normal distribution than other metrics

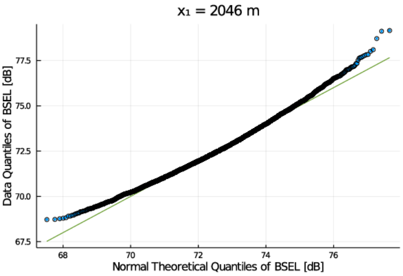


(a) $\sigma_u = 0.2$ m/s **(b)** $\sigma_u = 1.8$ m/s

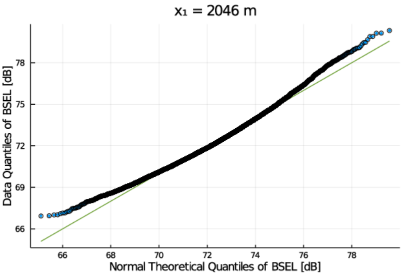
Figure 16: Quantile-Quantile plots of ASEL dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

BSEL C609 Waveform

- Similar trend to PL
- BSEL is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s



(b) $\sigma_u = 1.8$ m/s

Figure 17: Quantile-Quantile plots of BSEL dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

CSEL C609 Waveform

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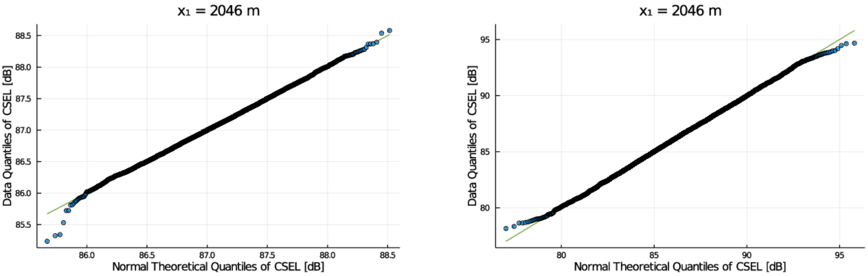
- Similar trend to PL
- CSEL is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s

(b) $\sigma_u = 1.8$ m/s

Figure 18: Quantile-Quantile plots of CSEL dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

DSEL C609 Waveform

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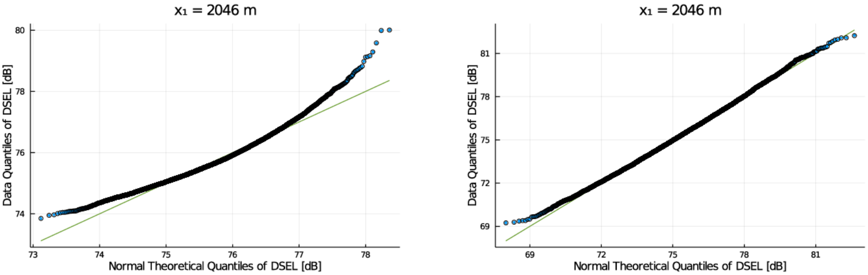
- DSEL is normal at medium turbulence intensities and skewed for low and high turbulence



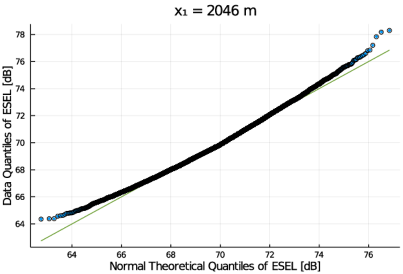
(a) $\sigma_u = 0.2$ m/s

(b) $\sigma_u = 1.8$ m/s

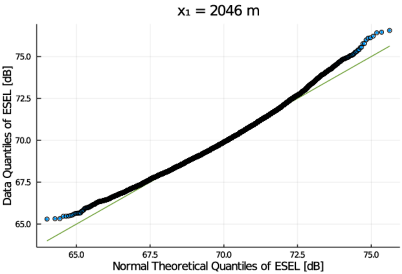
Figure 19: Quantile-Quantile plots of DSEL dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

ESEL C609 Waveform

- Similar trend to PL
- ESEL is skewed to the right at higher turbulence levels



(a) $\sigma_u = 0.2$ m/s

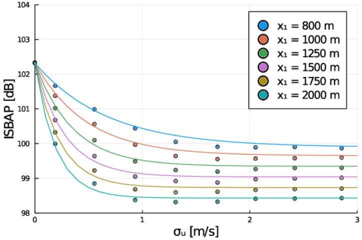


(b) $\sigma_u = 1.8$ m/s

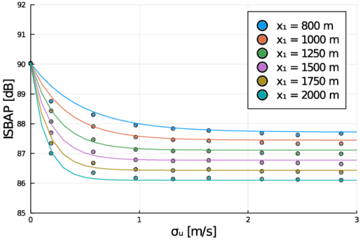
Figure 20: Quantile-Quantile plots of ESEL dB for the C609 Waveform at $x_1 = 2046$ m compared to a normal distribution (solid line) for two different turbulence intensities.

Average ISBAP I

- ISBAP follows same trend as PL with respect to increasing turbulence intensity
- Can be approximated with exponential decay model
- Not valid at low turbulence intensity for the C609 waveform



(a) N-wave



(b) Low Boom waveform

Figure 21: Average ISBAP as a function of σ_u for a) N-wave and b) C609, compared to exponential decay model.

Average ISBAP II

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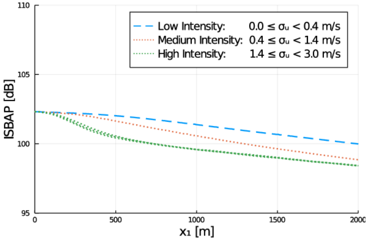
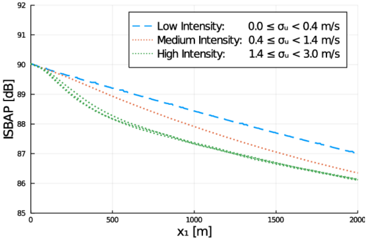
- Same trends as PL
- Approximately linear for $x_1 > 1000$ m for low and high turbulence intensities



(a) N-wave



(b) Low Boom waveform

Figure 22: Average ISBAP as a function of x_1 for a) N-wave and b) C609.

Average ASEL

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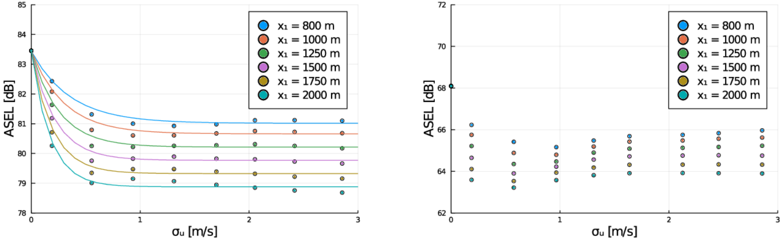
- ASEL follows same trend as PL for the N-wave
- For the C609 waveform, there is a significant drop in ASEL for medium turbulence levels
- At higher turbulence levels, the ASEL metric approaches a constant value



(a) N-wave **(b)** C609

Figure 23: Average ASEL as a function of σ_u for the N-wave and C609 waveform.

Average CSEL

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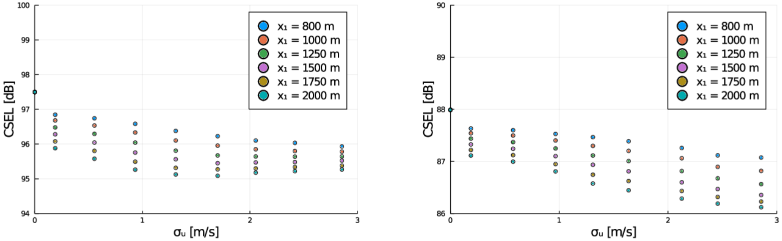
- CSEL does not follow PL or ISBAP trends
- For C609 waveform, CSEL is monotonically decreasing
- For the N-wave, CSEL approaches a constant value when $\sigma_u > 2.0$ m/s



(a) N-wave **(b)** C609

Figure 24: Average CSEL as a function of σ_u for the N-wave and C609 waveform.