Split-Step Simulations of Sonic Boom Propagation through Turbulence

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Background

- Shock waves propagate through atmosphere to ground
- Community annoyance from sonic booms prevents over land supersonic flight [1,2]
- Atmospheric turbulence leads to random distortions of the waveform
- Ray tracing codes do not account for turbulence [3,4,5,6]
- Recent push to incorporate atmospheric turbulence in predictions [7,8]

Fig 1. Illustration of sonic boom ray paths from the aircraft to the ground.

Fig 2. Measured sonic boom waveforms in low turbulence and high turbulence conditions. Adapted from [9].
Objectives

- **Overall objectives of work:**
  - Develop prediction tool for sonic boom propagation in atmospheric boundary layer (ABL)
  - Investigate turbulence effects on the waveform and loudness metrics
  - Predictions of sonic boom beyond the lateral cutoff in a turbulent atmosphere
- **In this presentation:**
  - Simulations of traditional N-wave and X-59 boom in homogeneous turbulence
  - Length scale for non-dimensionalization of results
  - Predictions of the caustic locations
  - Sonic boom overpressure predictions

Fig 3. Artist’s rendition of the NASA X-59 QueSST aircraft. Adapted from [1].
Method

- One-way propagation of acoustic pressure [8,10]:

\[
\frac{\partial^2 p}{\partial x_1 \partial \tau} = \mathcal{D}(p) + \mathcal{H}(p) + \mathcal{N}(p) + \mathcal{A}(p)
\]  

(1)

- \(\mathcal{D}\) (diffraction), \(\mathcal{H}\) (heterogeneities), \(\mathcal{N}\) (nonlinearities), \(\mathcal{A}\) (absorption)

- Split-step method [11]: compute each effect separately
  - Diffraction: angular spectrum method [12]
  - Heterogeneities: Crank-Nicolson scheme
  - Nonlinearities: Burgers solver [13]
  - Absorption: Absorption coefficient with relaxation [14]

- \(x_1\) is the propagation direction
- \(\Delta x_{1,2,3} = 2\) m

Fig 4. Computational domain.
Turbulence

- Energy spectrum: von Kármán model
- Frelich [15] method used to generate turbulent fields (Fourier synthesis technique)
- RMS velocity: $0.2 \leq u_{rms} \leq 3.0$ (m/s)
- Integral length scale: $L_f = 100$ m
- 25 fields per case, waveforms computed at 2500 microphone locations per case

Table 1. Simulation setpoints.

<table>
<thead>
<tr>
<th>Case</th>
<th>$u_{rms}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No turbulence</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>1.4</td>
</tr>
<tr>
<td>6</td>
<td>1.8</td>
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<tr>
<td>7</td>
<td>2.2</td>
</tr>
<tr>
<td>8</td>
<td>2.6</td>
</tr>
<tr>
<td>9</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Fig 5. Longitudinal velocity in the $x_3 = 0$ plane

Fig 6. Computed longitudinal and lateral correlations of the kinematic velocity fields compared to the von Kármán model

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Wavefront Folding

- Normalize propagation direction \( (x_1) \) by:

\[
\ell_f = \frac{C_1 L_f c_0 + u_{\text{rms}}}{2 u_{\text{rms}}} \quad (2)
\]

- Focal distance behind spherical lens of index:

\[
n = 1 + \frac{c_0}{u_{\text{rms}}} \quad (3)
\]

- Pierce and Maglieri [16] suggest radius of curvature of ripple caused by velocity perturbation

\[
R = \pi^2 L \frac{c_0}{\Delta u} \quad (4)
\]

- \( \ell_f \propto \frac{c_0}{u_{\text{rms}}} \) and \( R \propto \frac{c_0}{\Delta u} \)

- \( u_{\text{rms}} \) is the square root of the variance of fluctuations \( \sqrt{\mathbb{E}(\Delta u^2)} \)
Waveforms

- 12 kHz sampling frequency
- Initial waveforms shown alongside predictions at various $x_1 \ell_f^{-1}$ locations
- Spiking and rounding present in N-wave

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**Fig 8.** Initial N-wave shown with sample results at various locations along the propagation direction.

**Fig 9.** Initial X-S9 waveform shown with sample results at various locations along the propagation direction.
Caustic Locations

- Probability density function of the location of caustic locations ($x_{caust}$)
- Previous scaling (Kulkarny and White [17]) shown in Fig. 11
- Not collapsed for small $u_{rms}$
- Fig. 12 shows $\ell_f$ parameter collapses PDF for all $u_{rms}$

Fig. 10. N-wave overpressure for a single turbulence realization of case 9

Fig. 11. Probability density function of the location of the first caustic for 6 different N-wave cases scaled by the Kulkarny-White factor, compared to previous analytical result [17].

Fig. 12. Probability density function of the location of the first caustic for 6 different N-wave cases.

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Overpressure Statistics

- Average value of sonic boom overpressure ($\Delta p$) for both waveforms collapses when $x_1$ is scaled by $\ell_f$
- Linear decrease for X-59 waveform
- Maximum standard deviation of $\Delta p$ for N-wave double the maximum for X-59
- Approximately linear increase of $\sigma_{\Delta p}$ for $x_1 \ell_f^{-1} < 1$

Fig 13. Average value of sonic boom overpressure for N-wave and X-59 waveform.

Fig 14. Standard deviation of sonic boom overpressure for N-wave and X-59 waveform.
Summary and Conclusion

- Prediction tool has been developed to simulate nonlinear acoustic propagation in a turbulent medium
- Length scale proposed, based on analogy to focusing of light by spherical lens
- Most probable location of caustic formation for N-wave: \( x_1 \ell_f^{-1} = 1 \)
- Statistics of overpressure collapse along \( x_1 \ell_f^{-1} \)
- Indicates potential to parameterize \( \Delta p \) by \( u_{\text{rms}} \) and \( L_f \)
- Future work:
  - Simulations in inhomogeneous turbulence, representative of ABL
  - Simulations beyond the lateral cutoff into the shadow zone region
Thank you.

Questions?
References


