Split-Step Simulations of Sonic Boom Propagation through Turbulence

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Background

- Shock waves propagate through atmosphere to ground
- Community annoyance from sonic booms prevents over land supersonic flight [1,2]
- Atmospheric turbulence leads to random distortions of the waveform
- Ray tracing codes do not account for turbulence [3,4,5,6]
- Recent push to incorporate atmospheric turbulence in predictions [7,8]

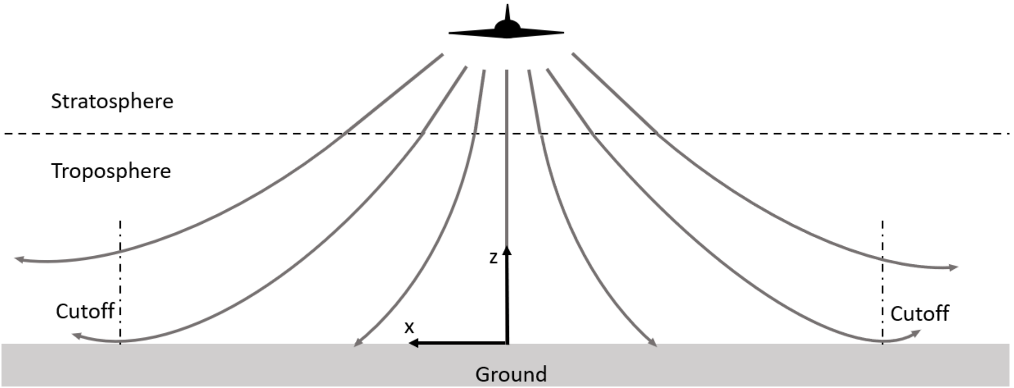

Fig 1. Illustration of sonic boom ray paths from the aircraft to the ground.

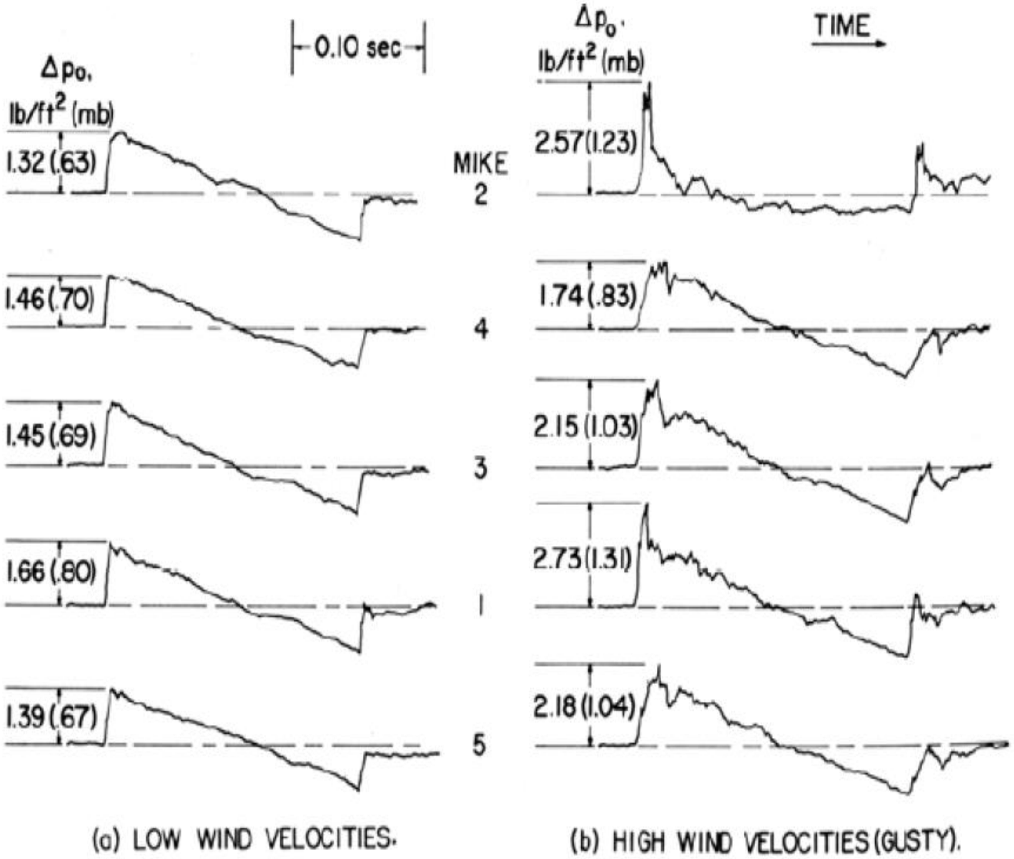

Fig 2. Measured sonic boom waveforms in low turbulence and high turbulence conditions. Adapted from [9].

Objectives

- Overall objectives of work:
  - Develop prediction tool for sonic boom propagation in atmospheric boundary layer (ABL)
  - Investigate turbulence effects on the waveform and loudness metrics
  - Predictions of sonic boom beyond the lateral cutoff in a turbulent atmosphere
- In this presentation:
  - Simulations of traditional N-wave and X-59 boom in homogeneous turbulence
  - Length scale for non-dimensionalization of results
  - Predictions of the caustic locations
  - Sonic boom overpressure predictions

Fig 3. Artist's rendition of the NASA X-59 QueSST aircraft. Adapted from [1].

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Method

- One-way propagation of acoustic pressure [8,10]:

$$\frac{\partial^2 p}{\partial x_1 \partial \tau} = \mathcal{D}(p) + \mathcal{H}(p) + \mathcal{N}(p) + \mathcal{A}(p) \qquad (1)$$

- $\mathcal{D}$ (diffraction), $\mathcal{H}$ (heterogeneities), $\mathcal{N}$ (nonlinearities), $\mathcal{A}$ (absorption)
- Split-step method [11]: compute each effect separately
  - Diffraction: angular spectrum method [12]
  - Heterogeneities: Crank-Nicolson scheme
  - Nonlinearities: Burgers solver [13]
  - Absorption: Absorption coefficient with relaxation [14]
- x_1 is the propagation direction
- $\Delta x_{1,2,3} = 2$ m

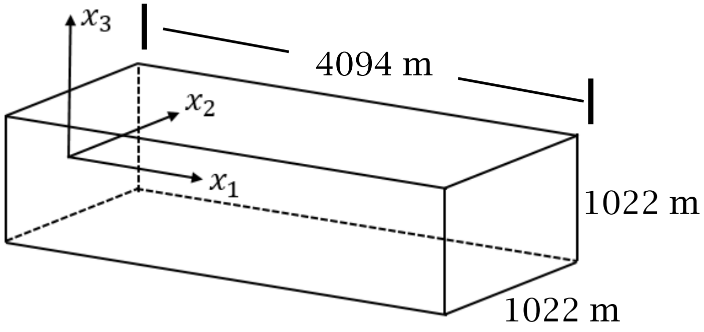

Fig 4. Computational domain.

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Turbulence

- Energy spectrum: von Kármán model
- Frelich [15] method used to generate turbulent fields (Fourier synthesis technique)
- RMS velocity: $0.2 \leq u_{rms} \leq 3.0$ (m/s)
- Integral length scale: $L_f = 100$ m
- 25 fields per case, waveforms computed at 2500 microphone locations per case

Table 1. Simulation setpoints.

Case	u_{rms} (m/s)
1	No turbulence
2	0.2
3	0.6
4	1.0
5	1.4
6	1.8
7	2.2
8	2.6
9	3.0

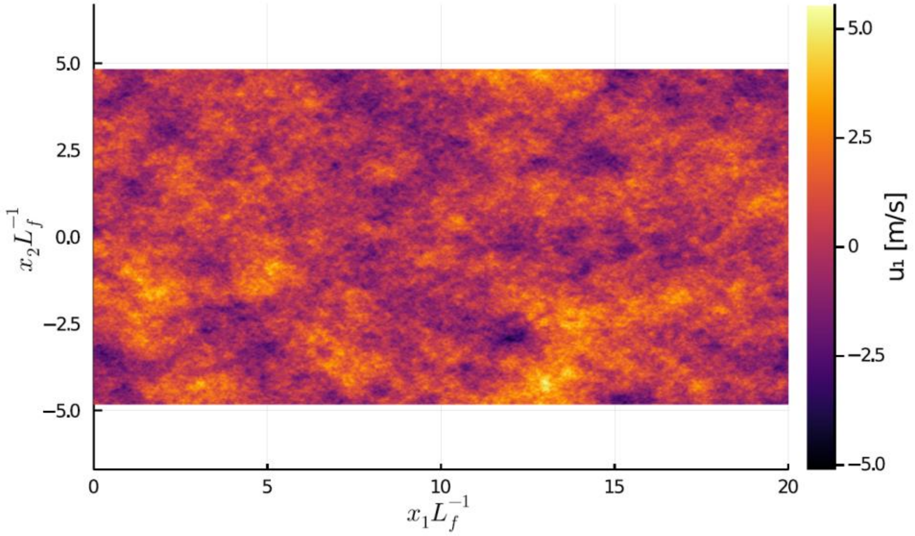

Fig 5. Longitudinal velocity in the $x_3 = 0$ plane

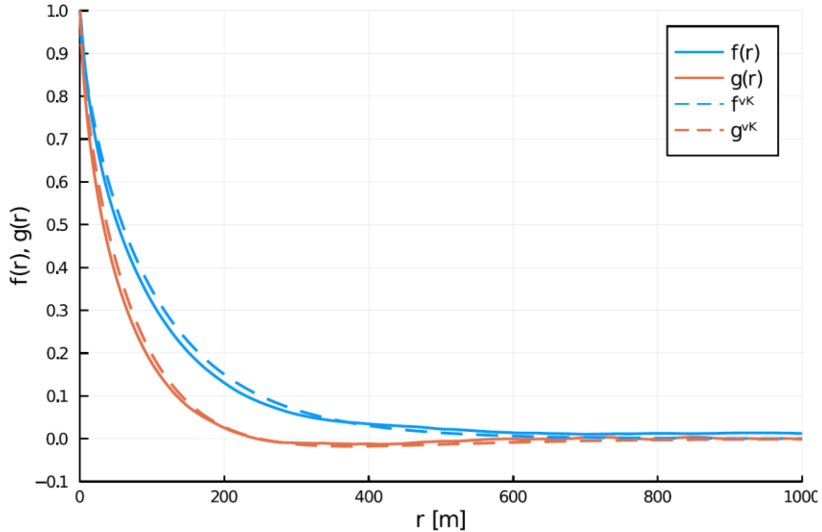

Fig 6. Computed longitudinal and lateral correlations of the kinematic velocity fields compared to the von Kármán model

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Wavefront Folding

- Normalize propagation direction (x_1) by:

$$\boxed{\ell_f = \frac{C_1 L_f}{2} \frac{c_0 + u_{rms}}{u_{rms}}} \quad (2)$$

- Focal distance behind spherical lens of index:

$$n = 1 + \frac{c_0}{u_{rms}} \quad (3)$$

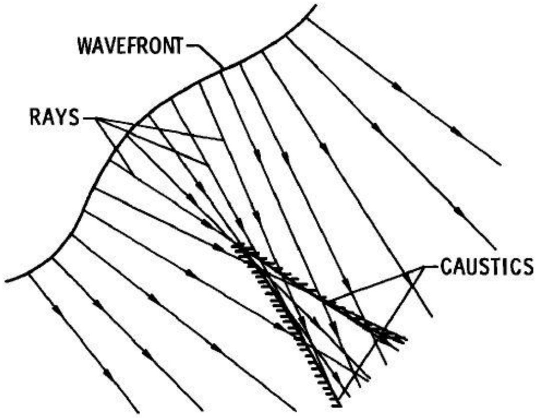

Fig 7. Illustration of wavefront focusing effect. Adapted from [16]

- Pierce and Maglieri [16] suggest radius of curvature of ripple caused by velocity perturbation

$$R = \pi^2 L \frac{c_0}{\Delta u} \quad (4)$$

- $\ell_f \propto \frac{c_0}{u_{rms}}$ and $R \propto \frac{c_0}{\Delta u}$
- u_{rms} is the square root of the variance of fluctuations $\sqrt{\mathbb{E}(\Delta u^2)}$

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Waveforms

- 12 kHz sampling frequency
- Initial waveforms shown alongside predictions at various $x_1\ell_f^{-1}$ locations
- Spiking and rounding present in N-wave

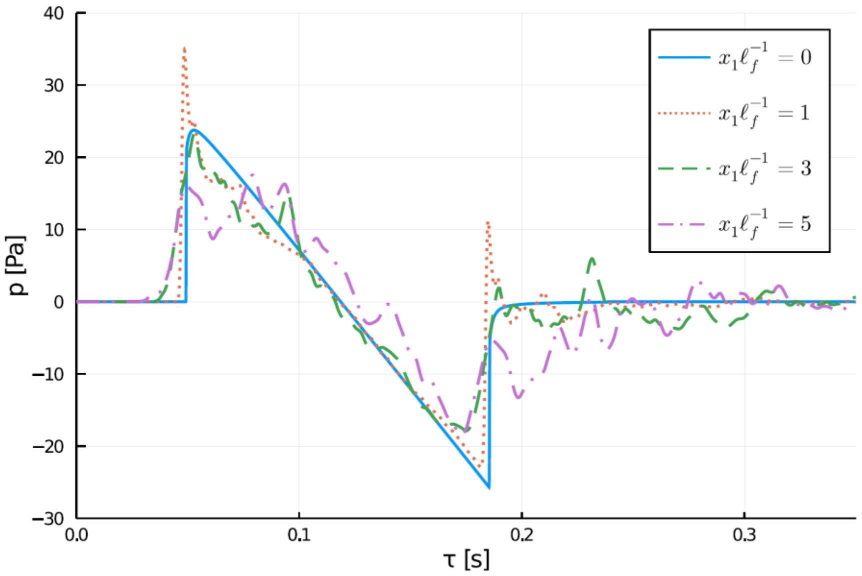

Fig 8. Initial N-wave shown with sample results at various locations along the propagation direction.

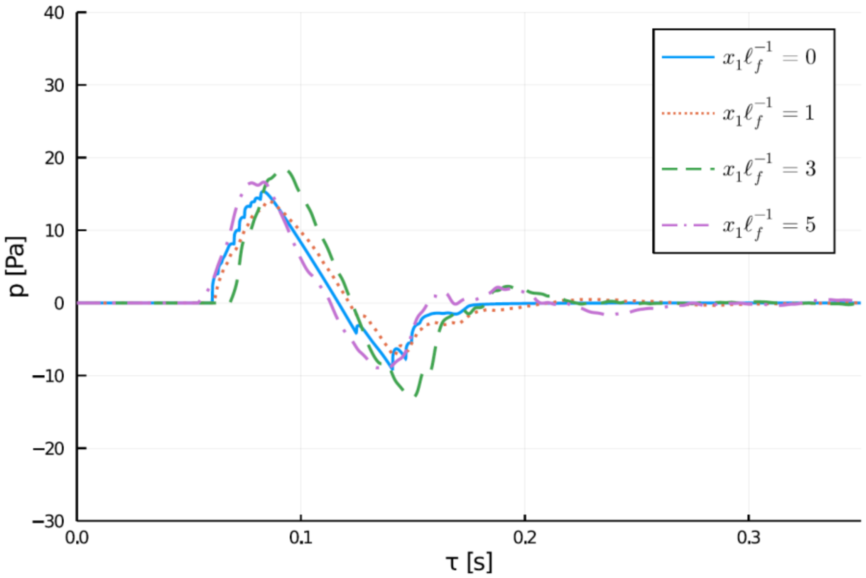

Fig 9. Initial X-59 waveform shown with sample results at various locations along the propagation direction.

Caustic Locations

- Probability density function of the location of caustic locations (x_{caust})
- Previous scaling (Kulkarny and White [17]) shown in Fig. 11
- Not collapsed for small u_{rms}
- Fig. 12 shows ℓ_f parameter collapses PDF for all u_{rms}

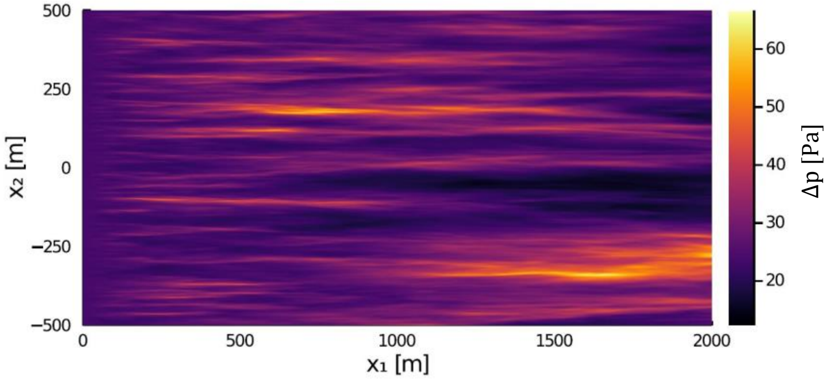

Fig 10. N-wave overpressure for a single turbulence realization of case 9

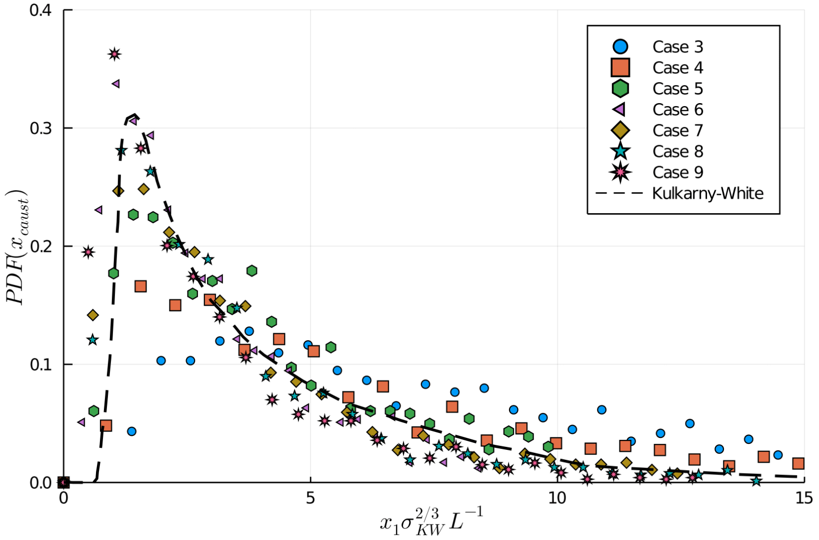

Fig 11. Probability density function of the location of the first caustic for 6 different N-wave cases scaled by the Kulkarny-White factor, compared to previous analytical result [17].

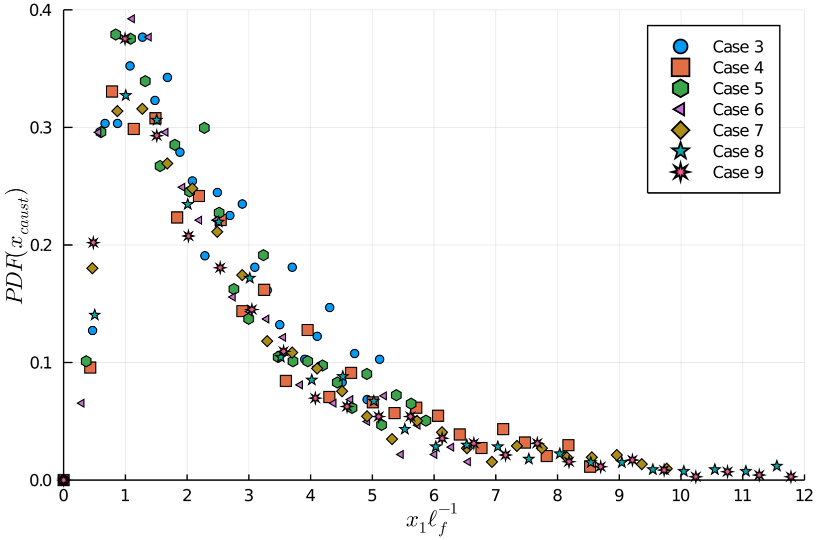

Fig 12. Probability density function of the location of the first caustic for 6 different N-wave cases.

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Overpressure Statistics

- Average value of sonic boom overpressure (Δp) for both waveforms collapses when x_1 is scaled by ℓ_f
- Linear decrease for X-59 waveform
- Maximum standard deviation of Δp for N-wave double the maximum for X-59
- Approximately linear increase of $\sigma_{\Delta p}$ for $x_1 \ell_f^{-1} < 1$

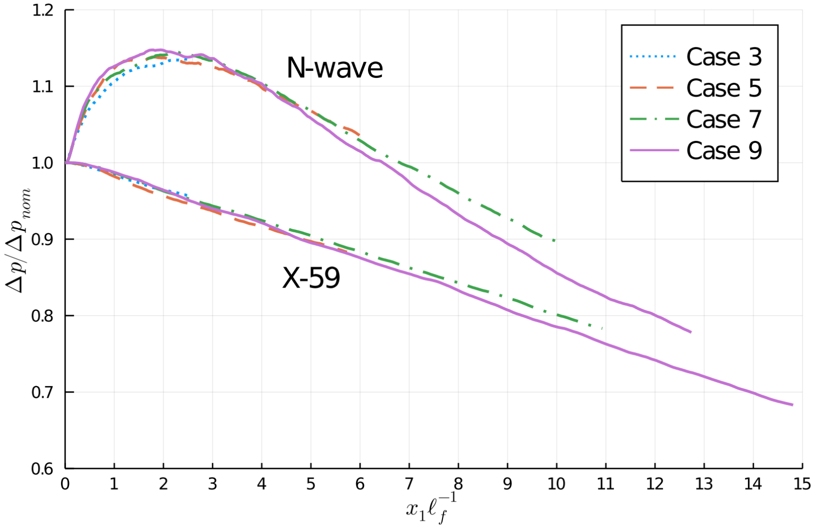

Fig 13. Average value of sonic boom overpressure for N-wave and X-59 waveform.

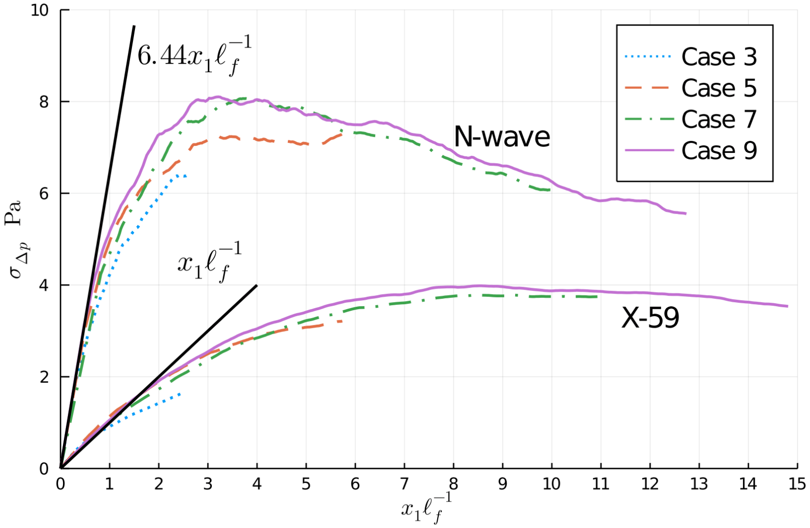

Fig 14. Standard deviation of sonic boom overpressure for N-wave and X-59 waveform.

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Summary and Conclusion

- Prediction tool has been developed to simulate nonlinear acoustic propagation in a turbulent medium
- Length scale proposed, based on analogy to focusing of light by spherical lens
- Most probable location of caustic formation for N-wave: $x_1 \ell_f^{-1} = 1$
- Statistics of overpressure collapse along $x_1 \ell_f^{-1}$
- Indicates potential to parameterize Δp by u_{rms} and L_f
- Future work:
  - Simulations in inhomogeneous turbulence, representative of ABL
  - Simulations beyond the lateral cutoff into the shadow zone region

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Thank you.

Questions?

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Prediction via Theory

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