

## Motivation

To aid in the design and development of hypersonic vehicles (where freestream Mach number,  $M_\infty$ , exceeds 5.0), it is important to quickly resolve the flow-field using fast flow solvers to minimize design time.

Previously, the inviscid Euler equations were sufficient for prediction of supersonic near-field for sonic boom propagation. However, the influence of viscosity, real gas effects, and chemical reactions in the hypersonic near-field cannot be ignored [1]. This computational framework serves to provide rapid hypersonic sonic boom predictive capability while taking into account these high energy physical effects.

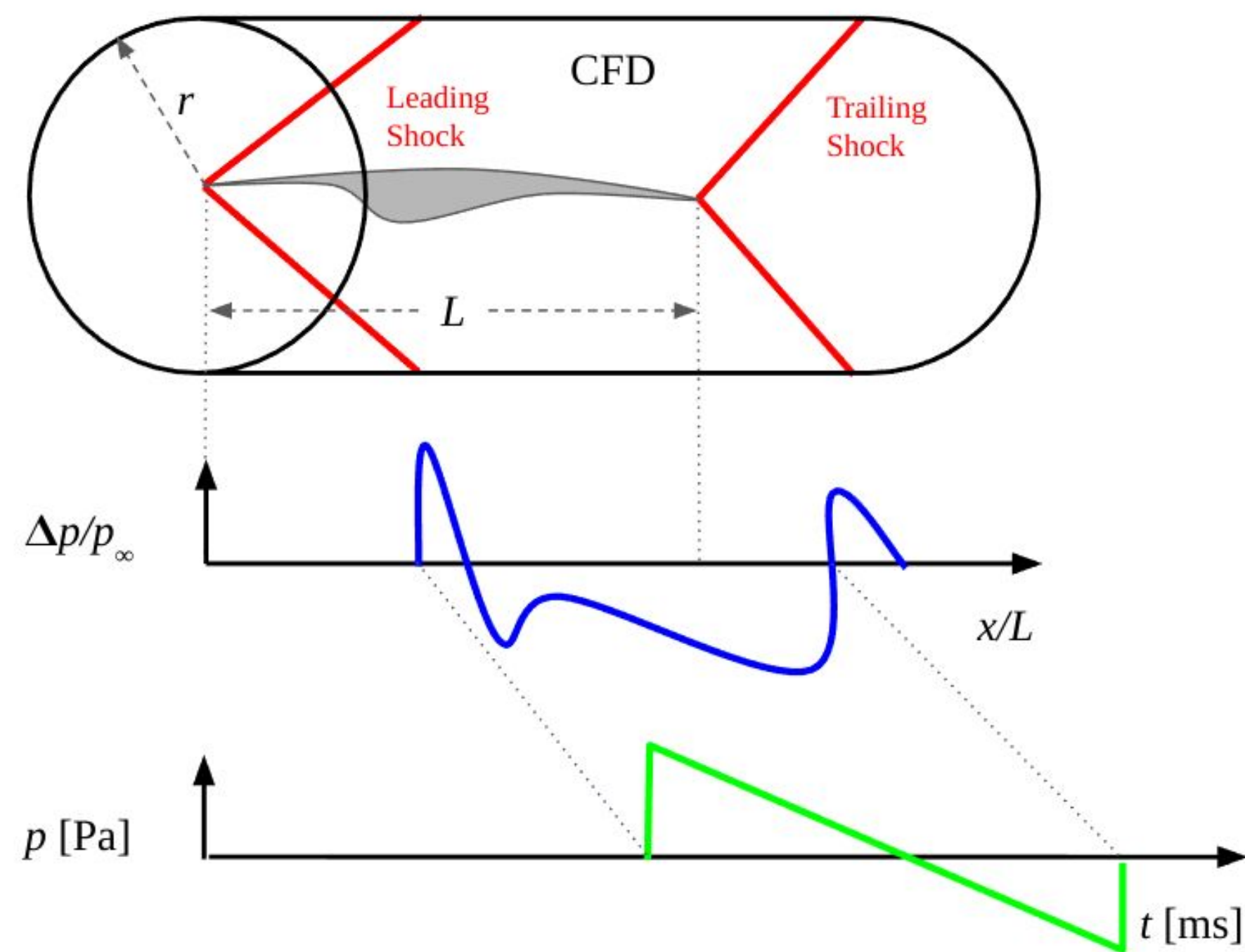
A successful, fully parabolized method for prediction of hypersonic sonic boom would reduce computation time in multi-disciplinary analysis and design (MDAO) frameworks.

## Methodology

Quickly predict hypersonic flow-field around vehicle.

Extract near-field source pressure.

Propagate near-field source pressure to the ground for sonic boom.



## Governing Equations

The Parabolized Navier Stokes (PNS) equations are derived from the full Navier-Stokes equations by assuming negligible unsteady and streamwise viscous derivatives [2].

These equations are suitable for hyperbolic-parabolic regions of flow: supersonic flow with exception of a subsonic, parabolic boundary layer. Solutions are marched in the upstream direction. The PNS equations, in Einstein notation, are:

$$\text{Mass: } \frac{\partial}{\partial x_k} (\rho u_k) = 0$$

$$\text{Momentum: } \frac{\partial}{\partial x_k} (\rho u_j u_k) = \frac{\partial}{\partial x_i} (-p \delta_{ij} + \tau_{ij}) + \rho f_j$$

$$\text{Energy: } \frac{\partial}{\partial x_k} (\rho u_k e_o) = -\frac{\partial}{\partial x_k} (u_k p) - \frac{\partial q_k}{\partial x_k} + \frac{\partial}{\partial x_k} (u_i \tau_{ij})$$

Hypersonic flow often induces high temperatures in the boundary layer and dissociates diatomic Nitrogen and Oxygen molecules. This phenomena is accounted for via closure of the species continuity equation [3].

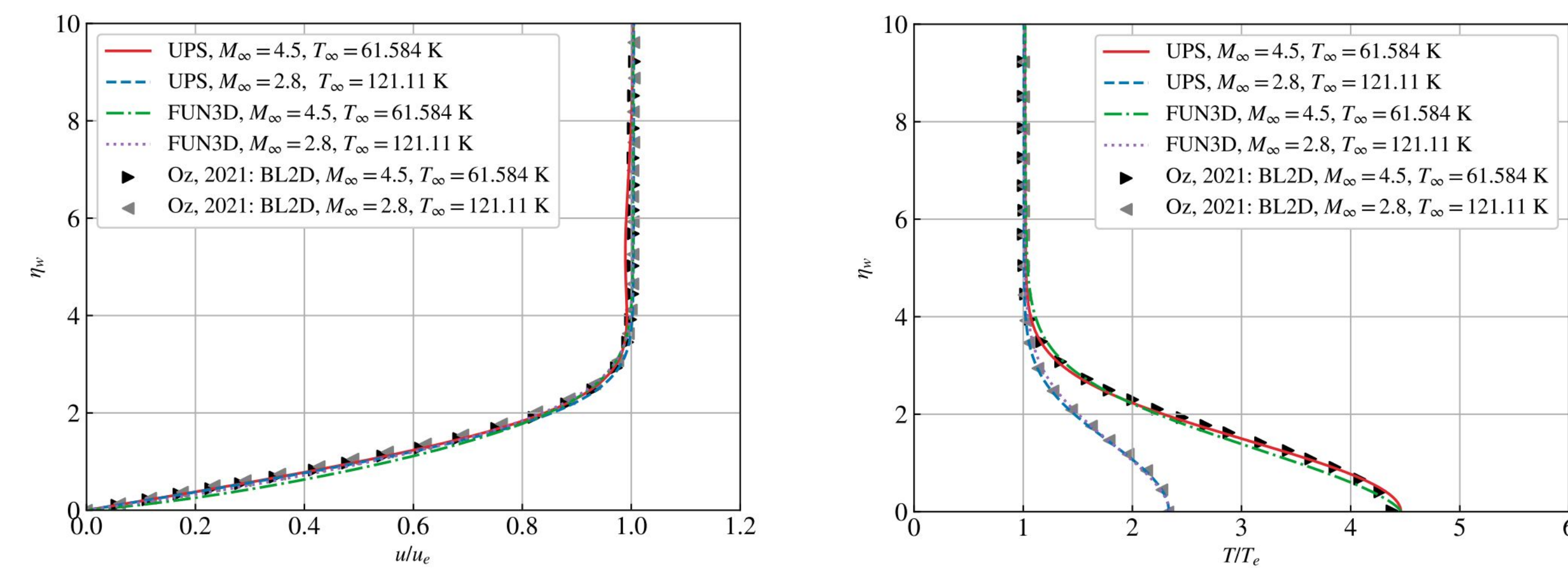
For propagation of the sonic boom to the ground, a source cylinder is extracted at a radius from the vehicle outside of the regions of vorticity. Sonic boom propagation is handled via Waveform Parameter Method (WPM). The waveform deformation equations that are solved in WPM are:

$$\frac{dm_i}{dt} = c_1 m_i^2 + c_2 m_i \quad \frac{d\Delta p_i}{dt} = \frac{1}{2} c_1 \Delta p_i (m_i + m_{i-1}) + c_2 \Delta p_i$$

$$\frac{d\lambda_i}{dt} = -\frac{1}{2} c_1 (\Delta p_i + \Delta p_{i+1}) - c_1 m_i \lambda_i$$

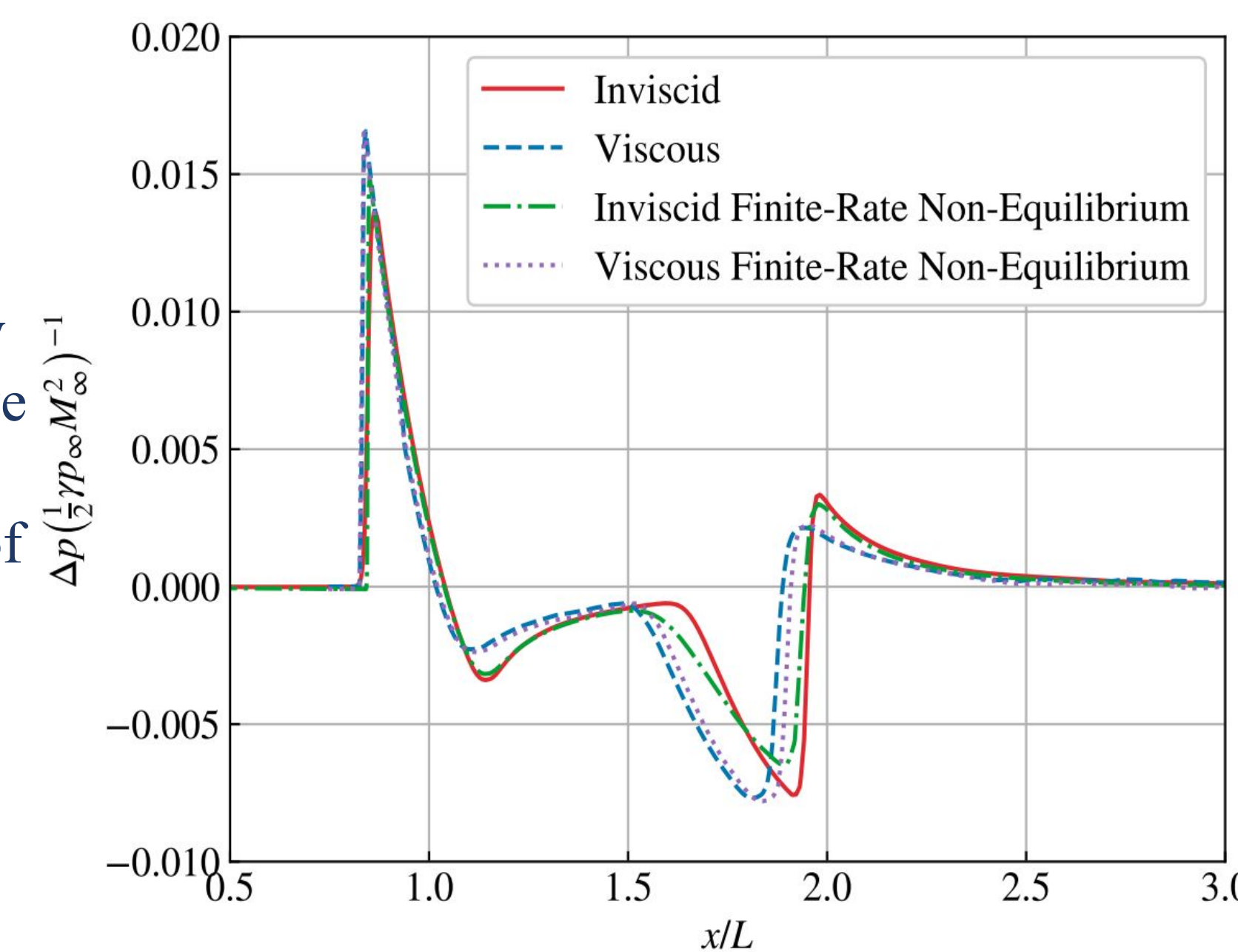
## Results

Boundary layer validation over a flat plate in comparison with NASA FUN3D and NASA BL2D for supersonic  $M_\infty = 2.8$  and 4.5



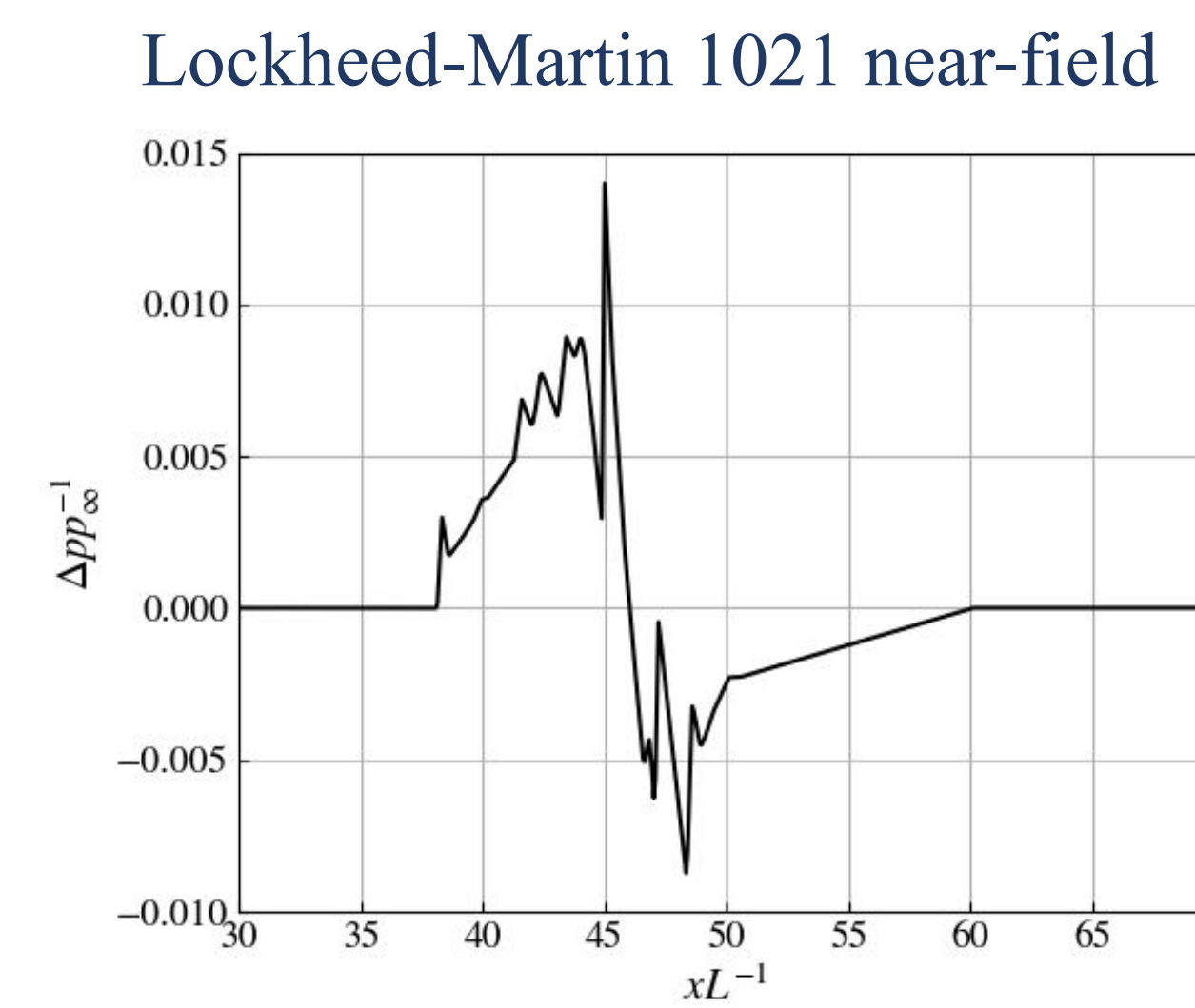
Cone-cylinder-cone (7.0 deg.) at  $M_\infty = 5.0$ . Near-field at  $rL^{-1} = 0.20$

- Increased overpressure prediction with viscous terms, 24% higher than inviscid
- Decreased trailing shock overpressure with viscous flow
- Decrease in leading shock wave position in the streamwise direction to due rapid growth of the compressible, hypersonic boundary layer
- Differing behavior throughout Prandtl-Meyer expansion region

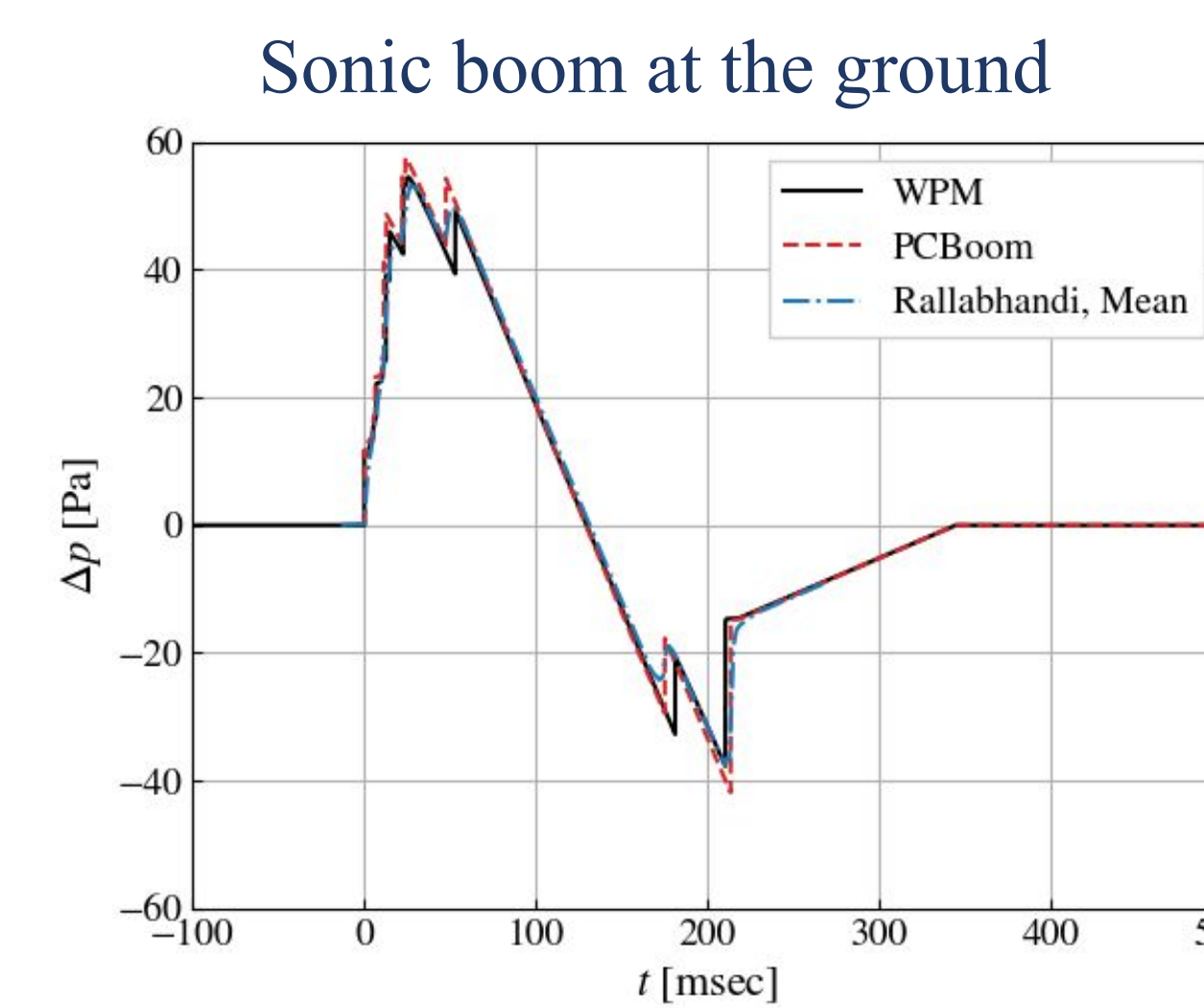


PCBoom validation with results from Second AIAA/NASA Sonic Boom Workshop and PCBoom [4]

- The Lockheed-Martin 1021 is a supersonic aircraft concept designed to minimize sonic boom signature.
- The near-field (top) is extracted via the pressures along a source cylinder outside the region of significant vorticity.
- As these waves propagate through the atmosphere, they coalesce and form stronger shock waves.
- The coalescence of these waves is heard at the ground as a sonic boom (bottom).

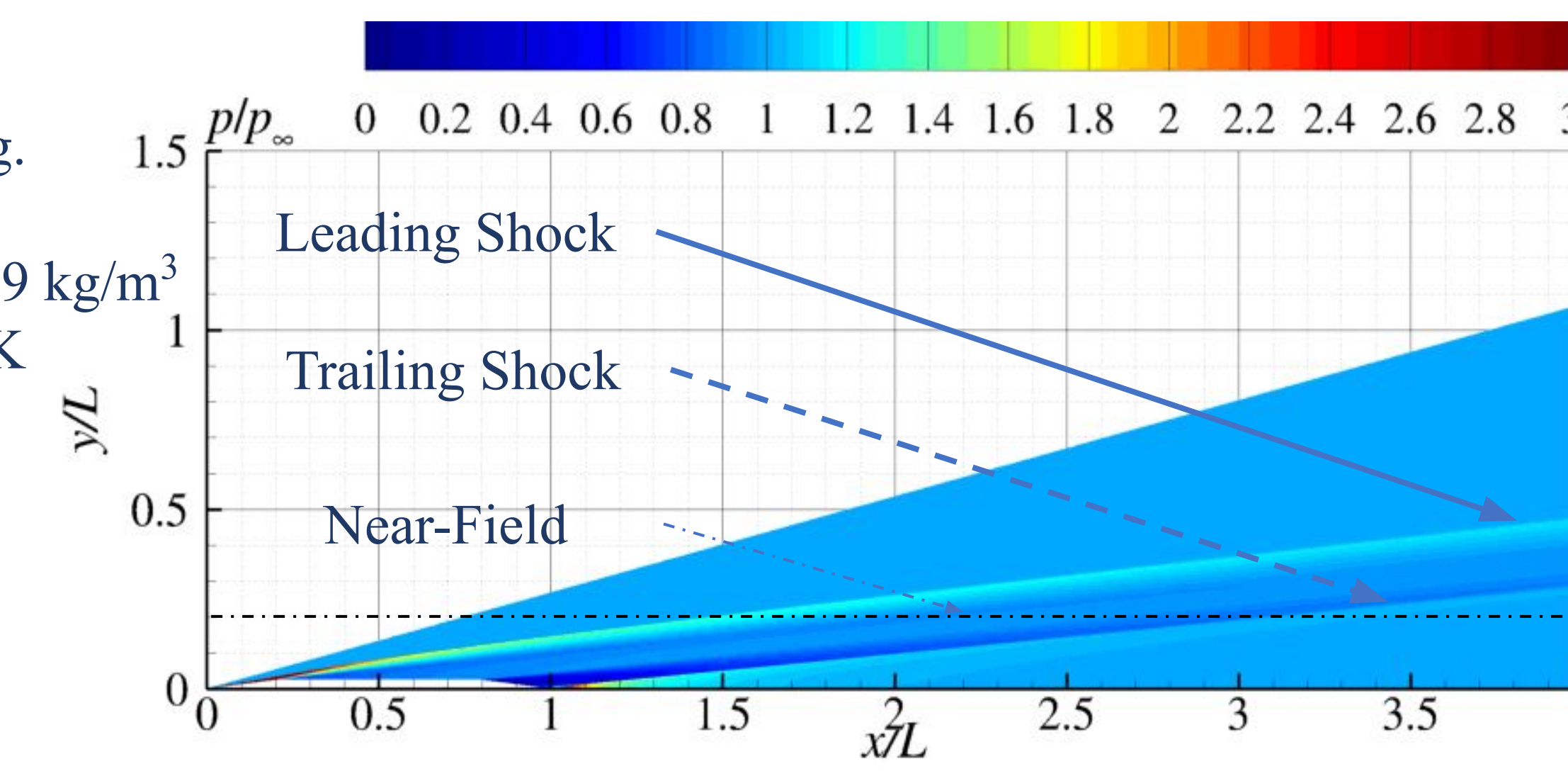


- $M_\infty = 1.6$
- Source cylinder:  $rL^{-1} = 3.1299$
- Vehicle length: 71.02 m
- Cruise altitude: 16764.0 m
- Ground reflection factor: 1.9
- Standard atmosphere



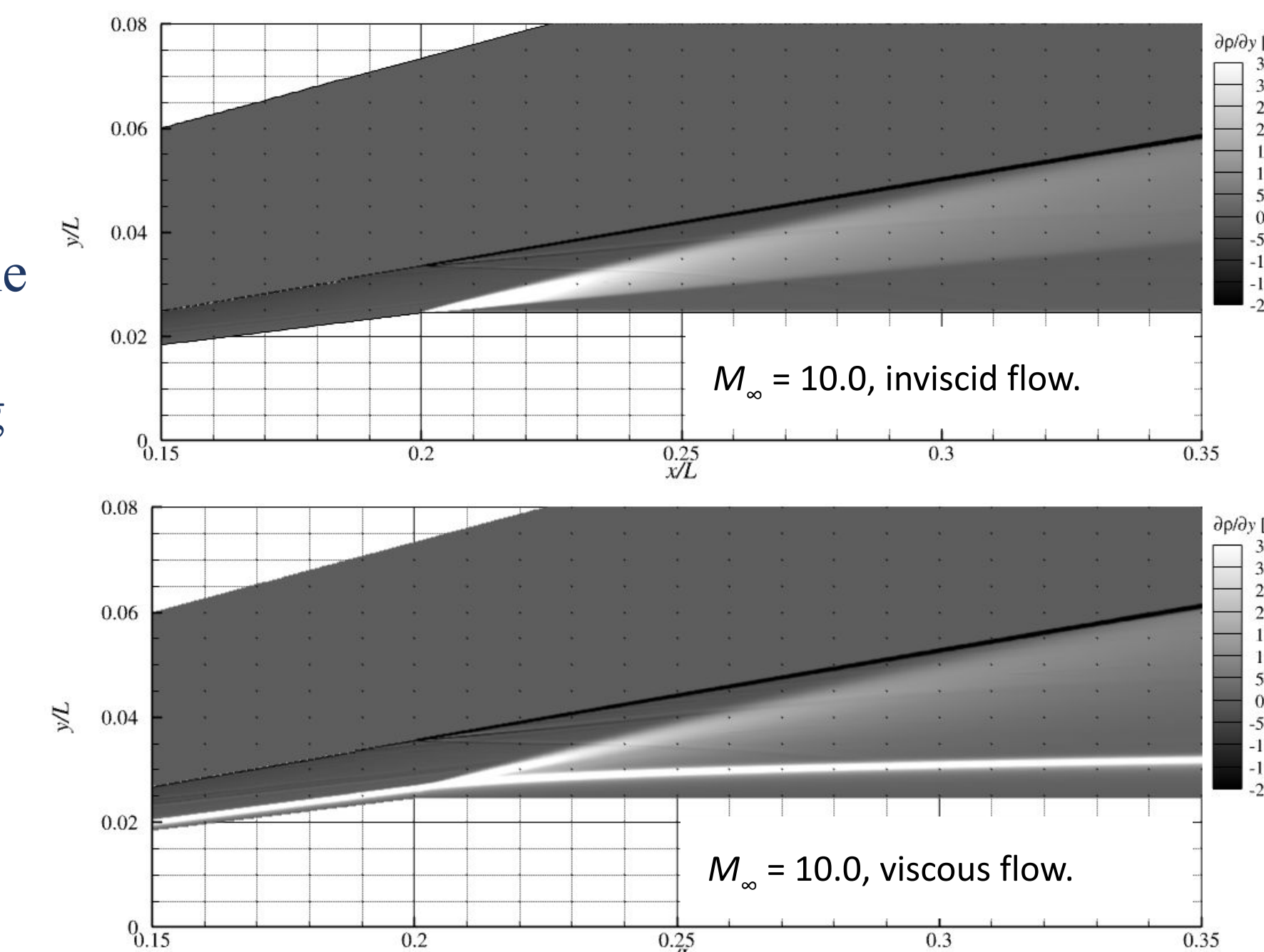
Cone-cylinder-cone (7.0 deg.) flow-field prediction

- $\theta_c = 7.0$  deg.
- $M_\infty = 10.0$
- $\rho_\infty = 0.0189$  kg/m<sup>3</sup>
- $T_\infty = 62.0$  K
- Inviscid



Viscous terms increase overpressure prediction by 8-12%, on average. Why is this?

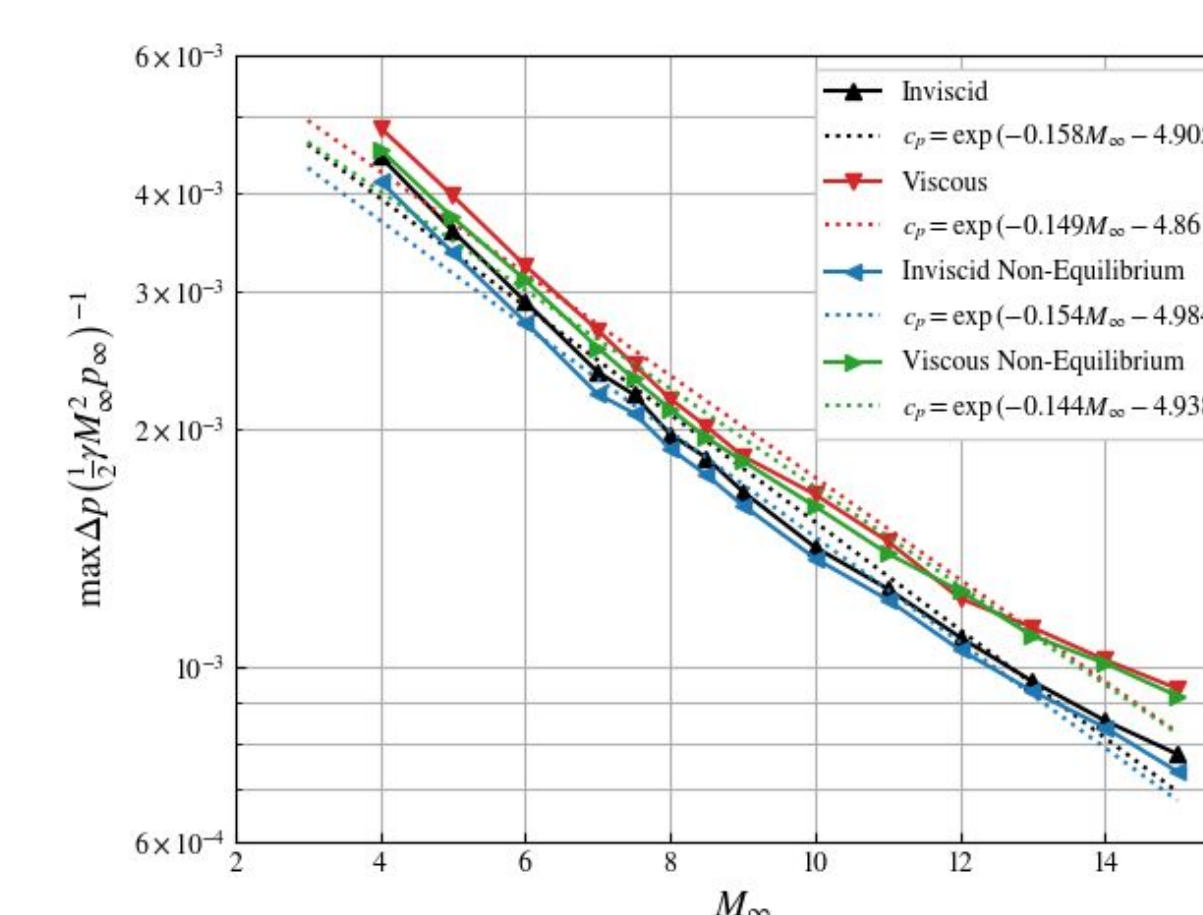
- For viscous flow, Prandtl-Meyer expansions are distributed across the boundary layer, rather than radiating from a single point.
- Shock waves are attenuated across a larger distance downstream.



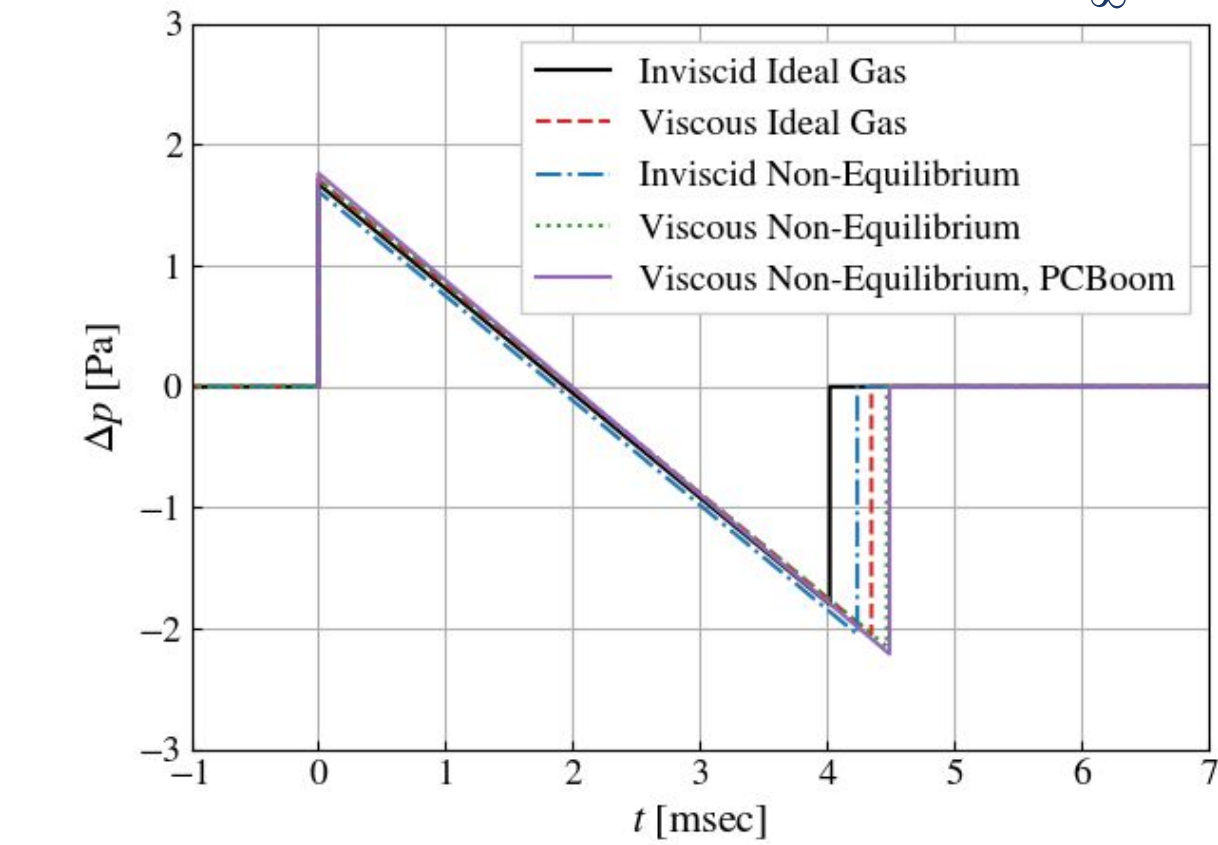
Sonic boom prediction for hypersonic Sears-Haack configuration

- The Sears-Haack body minimizes supersonic wave drag. It was discovered via the Prandtl-Glauert equation.
- Sonic boom signature at the ground is a perfect "N-wave".
- Near-field overpressure decreases log-linearly with  $M_\infty$ .
- Viscosity increases predicted overpressure. Non-equilibrium effects decrease overpressure and increase duration.

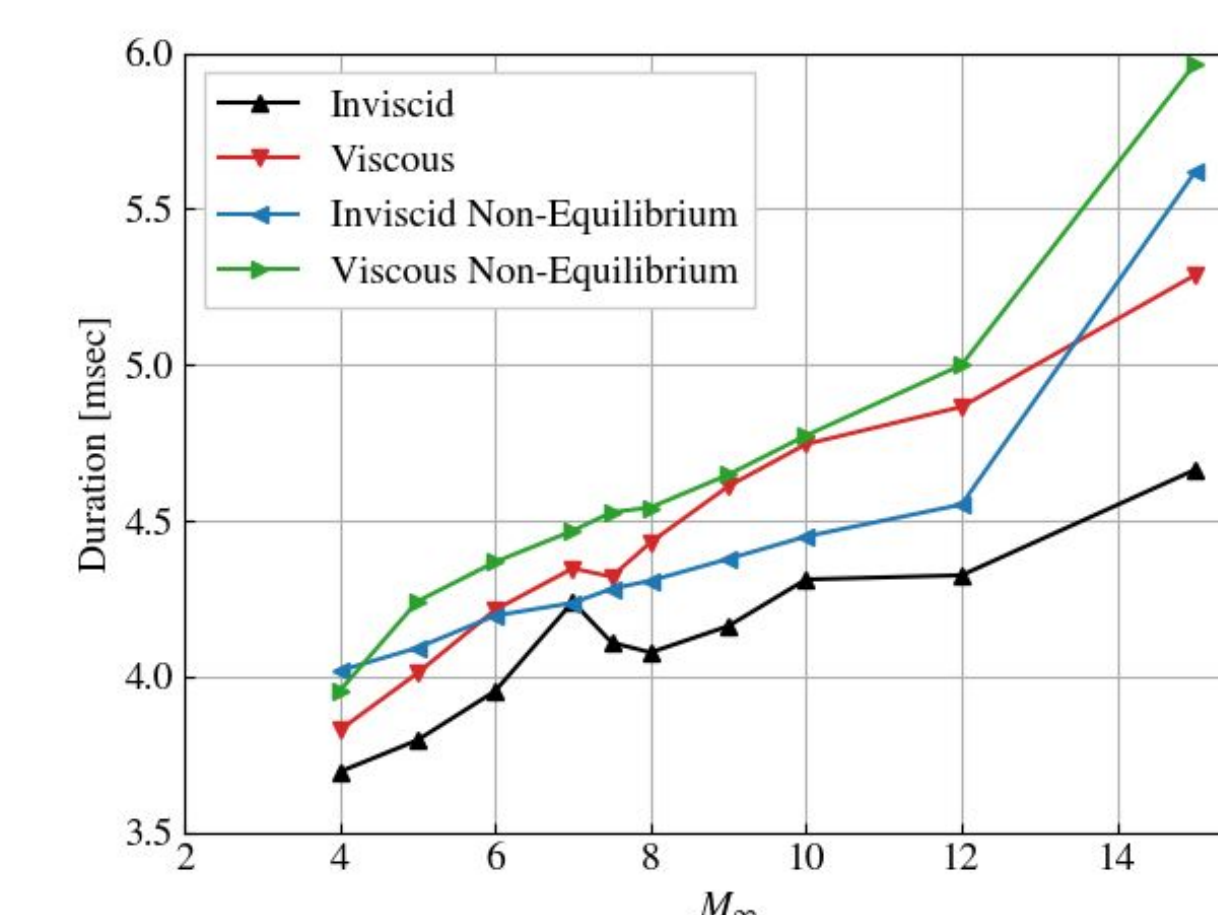
Near-field overpressure trends versus freestream Mach number



Propagation through a standard atmosphere from 15.8 km at  $M_\infty = 7.0$

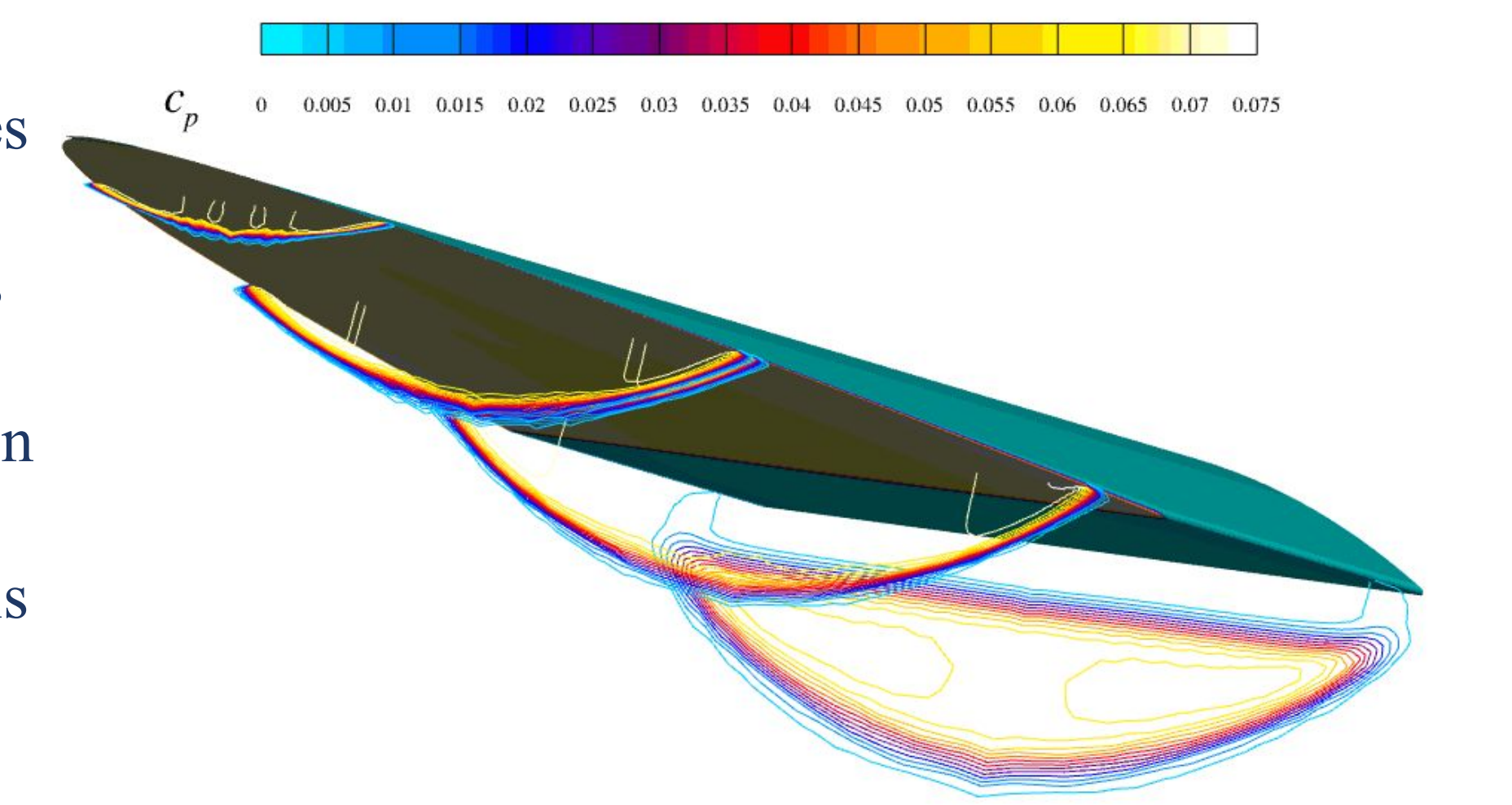


Sonic boom duration trends versus freestream Mach number



## Application to Power-Law Waverider Configuration

- Waveriders generate lift from shock waves close to the surface.
- The top surface does not produce a significant increase in pressure.
- The bottom surface is inclined by 7.0 deg. and produces significant lift.



## Conclusions

- A complete computational framework for hypersonic sonic boom prediction is developed.
- An Upwind Parabolized Navier-Stokes solver is used to predict the hypersonic near-field about slender-body vehicles and waveriders.
- Sonic boom predictions made with WPM show great agreement with PCBoom and mean data from the second AIAA Sonic Boom Workshop.
- Near-field overpressure decreases log-linearly with freestream Mach number.
- Inclusion of viscous terms increase overpressure whereas accounting non-equilibrium effects decreases overpressure.

## Future Work

- Expand real-gas parabolized method for generic waverider configurations at angles-of-attack for near-field prediction.
- Assess effects of inhomogeneous atmospheric turbulence on second mode instability via resolvent analysis with the parabolized stability equations.
- Incorporate two-equation Menter Shear-Stress-Transport turbulence model to account for turbulent kinetic energy and specific dissipation rate of the hypersonic flow-field.

## References

[1] King, C. and Miller, S. A. E., "Fully Parabolic Prediction of the Hypersonic Near-Field About Slender Axisymmetric Bodies," AIAA SciTech, National Harbor, MD, Jan. 23-27, AIAA Paper 2023-1424, 2023. doi: 10.2514/6.2023-1424.  
 [2] Lawrence, S. L., "Application of an Upwind Algorithm to the Parabolized Navier-Stokes Equations," Ph.D. dissertation, Iowa State University, 1987.  
 [3] Tannehill, J., Buelow, P., Ievalts, J., and Lawrence, S., "A Three-Dimensional Upwind Parabolized Navier-Stokes Code for Real Gas Flows," 24th Thermophysics Conference, AIAA Paper 1989-1651, 1989. doi:10.2514/6.1989-1651.  
 [4] Rallabhandi, S., and Loubeau, A., "Summary of Propagation Cases of the Second AIAA Sonic Boom Prediction Workshop," Journal of Aircraft, Vol. 56, No. 3, 2019, pp. 876–895. doi:10.2514/1.C034805

## Acknowledgement

Research was sponsored by the Defense Advanced Research Project Agency (DARPA) and the Army Research Office and was accomplished under Grant Number W911NF-21-1-0342. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Office or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation herein.