A New Course: Navier-Stokes Equations with a Historical Perspective

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Outline

- Motivation and Desired Outcome
- Objectives and Overview
- Course outline
- Class philosophy
- Handouts
- Historical Figures
- Online materials

Motivation and Desired Outcome

- Problem – Much research (analytical) involving Navier-Stokes equations occurs in mathematics departments. How do we convey these developments to the greater fluid dynamics community and also apply important lessons to practical flow problems?
- Solution – Bridge the gap between the mathematics and engineering communities by examining current literature and forming a practical course for graduate students. Make it available to the community.
- Benefit – Practical and beneficial understanding of current analysis of NSE, greater interest of students in the subject, and less mistakes within the CFD community.

Major Class Objectives

The student will understand the history, physical meaning, and contemporary challenges within the field of theoretical fluid dynamics and turbulence.

Course Overview

Course Description - Course catalogue: Navier-Stokes Equations (NSE) History, derivation, physical meaning, classical solutions, stability, dynamical systems, existence, uniqueness, regularity, scales, ladder results, dissipation rates, Serrin?s blowup, capacitary approaches, mild solutions (Lebesgue, Sobolev, Besov, Morrey, BMO, Koch, Tataru), weak solutions, stochastic NSE, the Russian school, and invariant measures..

Course Outline I

- Introduction
  - Introduction, Syllabus, Outline of Course, Clay, Interested in Solutions
- History
  - Overview
  - Detail
- Mathematical Review – Review, Notation
- Derivation of NS
- Physical Meaning – Physical Meaning of NS
- Classical Solutions – Classical solutions of the NS
- Stability – Introduction to linear and nonlinear stability
- Dynamical Dynamical Systems
- Intro Exist Unique Regularity
  - Existence, Uniqueness, and Regularity
  - Regularity and intro to length scales for the 2D and 3D NSE

Course Outline II

- Ladder – Results
- Dissipation Rates – Energy dissipation rates of Fourier spectra - bounded flow
- Blowup – Criteria, Serrin's criterion
- Capacitary approach of the NSE integral equations
- Differential and integral NSE
- Mild solutions
  - Lebesgue / Sobolev spaces
  - Besov or Morrey spaces
  - BMO^{-1} and Koch and Tataru theorem
- Leray's weak solutions
- Stochastics
  - Statistics and Derivation of Stochastic NSE
  - Probability and Statistical Theory of Turbulence
  - Invariant measures and PDF
  - Existence theory of swirling flow – existence of stochastic NSE

What is this Class? What is it not?

- What this class is
  - Examination of classical and contemporary approaches for solutions and analysis of NSE
  - How are these methods related to societal problems – and how can they be moved to predictive methods
  - A lot of effort on the part of students to understand the material
  - A lot of material
- What this class is not
  - Derivation of trivial solutions of NSE
    - Example – laminar flows with simplified boundary conditions (BC)s
  - Easy to understand
  - Review of earlier mathematics or fluids classes in CFD, turbulence, or mathematics
    - If you don't understand something take the initiative and look it up

Comment on Handouts

- Before every class the handout is posted on the website
- Please print the slides or bring your digital electric writing device
- As we go through the class fill in the blanks
- Make whatever notes you wish on the slides before, during, or after class
- Example
  - The following equation is included in the handout
    $y = mx + b$
  - The following equation is filled in by students

- This method was used by my advisor during graduate school and is effective

Example Historical Figures

Jean-Baptiste le Rond d'Alembert

16 November 1717 – 29 October 1783, French

"time destroyed all models which the ancients may have left us in this genre"

- Mathematician, physicist, philosopher, and music theorist
- d'Alembert solution to wave equation
- Mother left him on the steps of the Saint-Jean-le-Rond de Paris church. Named after the patron saint of the church. Education secretly paid for by his father of royalty.
- 1740 *Mémoire sur la réfraction des corps solides*, theoretically explained refraction
- 1752, wrote D'Alembert's paradox: that the drag on a body immersed in an inviscid, incompressible fluid is zero.
- Participant in several Parisian salons
- A known 'unbeliever,' was buried in a common unmarked grave.

Example Historical Figures

Marie-Sophie Germain
1 April 1776– 27 June 1831, French

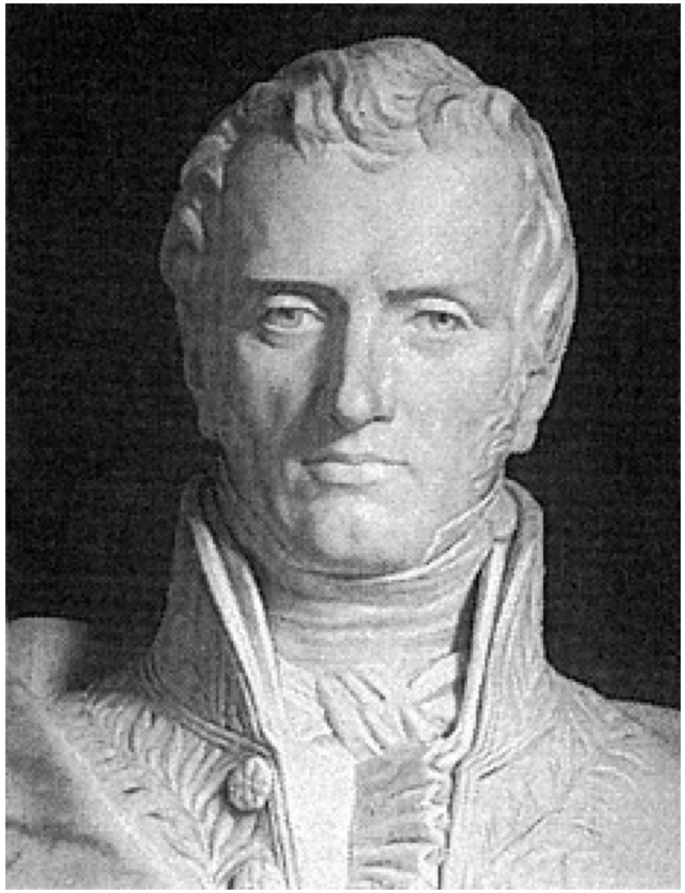
- French mathematician, physicist, and philosopher
- Began her education from reading books by Euler in her father's library, despite opposition from her parents
- When Germain was 18 she obtained lecture notes from Ecole Polytechnique and sent her work to Lagrange under a male name, he went on to become her mentor
- Germain entered a paper into a contest held by the French Academy of Sciences to give a mathematical theory of the vibration of an elastic surface – after two attempts her third paper was successful and she became the first woman to win the prize
- Worked on Fermat's Last Theorem and provided a proof to the first case of the theorem for all odd primes less than 100
- Also worked on philosophy and psychology and argued there is no difference between science and the humanities
- Sophie Germain prize established in 2003 to honor a French mathematician for research in the foundations of mathematics

Example Historical Figures

Claude Louis Marie Henri Navier

10 February 1785 – 21 August 1836, French

- Engineer and physicist who specialized in mechanics
- Known for Navier-Stokes equations
- École polytechnique, and in 1804 continued his studies at the École Nationale des Ponts et Chaussées, from which he graduated in 1806
- Construction of bridges at Choisy, Asnières and Argenteuil
- Professor of calculus and mechanics at the École Polytechnique
- Named on Eiffel tower



Example Historical Figures

George Gabriel Stokes

13 August 1819 – 1 February 1903, British

- Physicist and mathematician, fluids, optics
- Navier-Stokes equations
- Stokes spent all of his career at the University of Cambridge, where he served as Lucasian Professor of Mathematics
- President Royal Society
- President of the Victoria Institute, which had been founded to defend evangelical Christian principles against challenges from the new sciences, especially the Darwinian theory of biological evolution
- Married Mary Susanna Robinson, daughter of the Rev Thomas Romney Robinson, 5 children

Class Outcome – Feedback and Evaluations

- 81.25% (responded: 26, enrolled: 32)
- COVID – Difficult transition online within 3 classes
  - I think we all remember this difficult time in our class transition

Question	University Core	Percentages				
Number	Questions	(1)	(2)	(3)	(4)	(5)
Q10	The course fostered regular interaction between student and instructor.	0	6.25	6.25	12.5	56.25
Q11	Course activities and assignments improved my ability to analyze, solve problems, and/or think critically.	0	3.13	9.38	25	43.75
Q12	Overall, this course was a valuable educational experience.	0	3.13	9.38	18.75	50

Figure 1: Evaluation summary of class.

Term Paper

- Term paper
- Students choose their own topic or pick from a pre-approved list, but can't overlap with other students
- First assignment – basic background review and term paper proposal
- Second assignment – draft abstract and detailed outline (3 pages)
- Final assignment – ten page paper with equations, plus additional pages for figures and references

Materials Available Online

- Comprehensive set of notes that covers the state-of-the-art in understanding the Navier-Stokes equations.
- The course has approximately
  - 1768 slides
  - over 2000 unique equations
  - 92 unique historical figures with pictures and fun facts
  - and many interspersed quotes of famous researchers
- Two years to create!

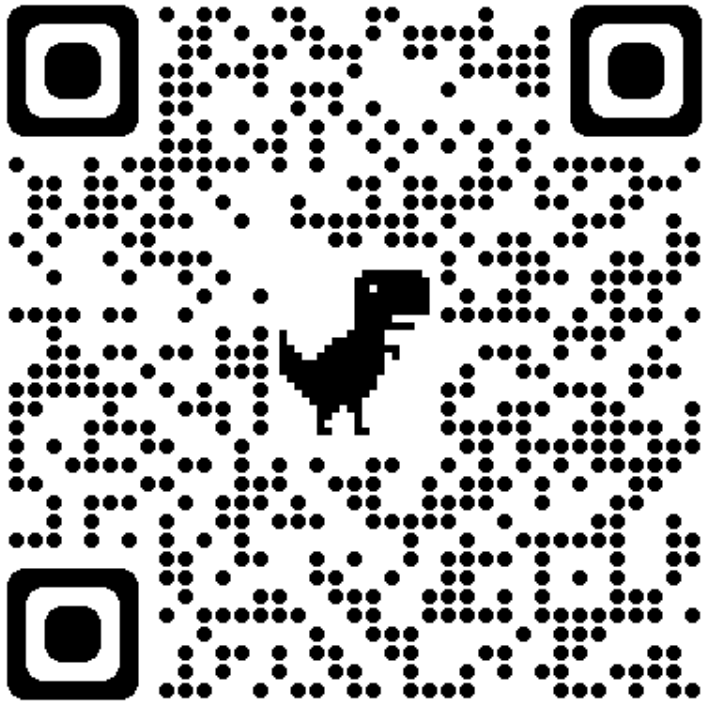


Figure 2: https://saemiller.com/navier-stokes/

Summary and Concluding Remarks

- Created a contemporary class that is available for engineering students to understand modern problems in Navier-Stokes research
- Available to all
- Future
  - Record these lectures for online viewing - free of charge for anyone wanting to view

Thank You