Contemporary Challenge of Connecting Turbulence Models with Time and Spectral Analytical Acoustic Sources

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Outline

- Introduction
- Typical RANS models for Aeroacoustics
- Traditional Connection
- LES / FW-H Approach and its Major Drawbacks
- Analytical Acoustic Sources from Turbulence
  - Small-Scale Turbulence
  - Shockwave Shear-Layer Interaction
  - Large-Scale Coherent Turbulence, an enigma in the acoustics community
- Connection to Wavenumber Spectra
  - Noise is the fingerprint of turbulence
- Disconnect Between Turbulence Closure and Aeroacoustics
- Summary

Introduction

- The turbulence modeling community and the aeroacoustics community generally create separate prediction models
  - Turbulence modeling community goal is to predict accurate turbulent flow statistics
  - Acoustic community wishes to predict accurate radiated noise
- Outcome of both are relatable through semi-empirical means
  - Usually up to acoustics community to make the connection
- Approach of LES can be used with Ffowcs-Williams and Hawking approach
  - We learn nothing about the sources of noise and the sources of FWH are analogies of the actual source of turbulent noise
  - Huge computational expense and does not make use of typical large RANS databases available within industry

RANS I

Reynolds-averaged Navier-Stokes equation (RANS)

$$\rho \frac{\partial U_i}{\partial t} + \rho U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\mu S_{ji} - \rho \overline{u_j' u_i'}\right) \tag{1}$$

Reynolds stress equation

$$\begin{aligned} \frac{D\overline{u_i u_j}}{Dt} = \frac{\partial}{\partial X_l}\left(-\overline{u_i u_j u_l} - \overline{\frac{p}{\rho}\left(\delta_{jl} u_i + \delta_{il} u_j\right)} + \nu \frac{\partial \overline{u_i u_j}}{\partial X_l}\right) \\ -\left(\overline{u_i u_l}\frac{\partial U_j}{\partial X_l} + \overline{u_j u_l}\frac{\partial U_i}{\partial X_l}\right) - 2v\overline{\frac{\partial u_i}{\partial X_l}\frac{\partial u_j}{\partial X_l}} + \overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)} \end{aligned} \tag{2}$$

RANS I

ϵ equation is

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial X_l}\left(-\overline{\epsilon' u_l} - \frac{2\nu}{\rho}\overline{\frac{\partial u_l}{\partial X_j}\frac{\partial p}{\partial X_j}} + \nu\frac{\partial \epsilon}{\partial X_l}\right) - 2\nu\overline{u_l\frac{\partial u_i}{\partial X_j}}\frac{\partial^2 U_i}{\partial X_l \partial X_j}$$
$$-2\nu\frac{\partial U_i}{\partial X_j}\left(\overline{\frac{\partial u_l}{\partial X_i}\frac{\partial u_l}{\partial X_j}} + \overline{\frac{\partial u_i}{\partial X_l}\frac{\partial u_j}{\partial X_l}}\right) \quad (3)$$
$$-2\nu\overline{\frac{\partial u_i}{\partial X_j}\frac{\partial u_i}{\partial X_l}\frac{\partial u_j}{\partial X_l}} - 2\overline{\left(\nu\frac{\partial^2 u_i}{\partial X_l \partial X_l}\right)^2}$$

TKE equation of the mean motion

$$\frac{D}{Dt}\left(\bar{\varrho}\frac{\tilde{u}_i\tilde{u}_i}{2}\right) = -\tilde{u}_i\frac{\partial \bar{p}}{\partial x_i} + \tilde{u}_i\frac{\partial \bar{\tau}_{ik}}{\partial x_k} - \tilde{u}_i\frac{\partial}{\partial x_k}\left(\overline{\varrho u_i' u_k'}\right) \quad (4)$$

Outcome of RANS CFD Simulations

Typical RANS Output

- Geometry
- Computational grid
- Mean variables
- Two-equation models
  - Usually k and ω or ϵ
- Reynolds Stress Models – Reynolds stresses and ϵ or ω

Acoustic Analogy – A Traditional Path via Lighthill

- Rearrangement of continuity and momentum into wave equation with unknown left and right hand sides
- Sources are analogues of actual sources, which is why it is called the acoustic analogy

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{5}$$

and

$$T_{ij} = \rho v_i v_j + p_{ij} - c_\infty^2 \rho \delta_{ij} \tag{6}$$

Figure 1: Sir James Lighthill.

Traditional Way of Connecting Acoustic Sources to RANS

Acoustic source models depend on scales to estimate two-point correlations

Length scale

$$l_x(\boldsymbol{y}) = c_l K(\boldsymbol{y})^{3/2} / \epsilon(\boldsymbol{y}) \tag{7}$$

Time scale

$$\tau_s(\boldsymbol{y}) = c_\tau K(\boldsymbol{y}) / \epsilon(\boldsymbol{y}) \tag{8}$$

Velocity scale

$$u_s(\boldsymbol{y}) = u_s \sqrt{2K(\boldsymbol{y})/3} \tag{9}$$

Connecting Lighthill's Model to RANS

$$\overline{\frac{\partial^2 T_{ij}}{\partial \tau^2} \frac{\partial^2 T'_{lm}}{\partial \tau'^2}} \approx \frac{\partial^4 \boldsymbol{R}_{ijlm}}{\partial \tau^4}. \tag{10}$$

and

$$\boldsymbol{R}_{ijlm} = \overline{\rho}\ \overline{\rho}' \left(\overline{u_i u_j}\ \overline{u'_l u'_m} \right) R, \tag{11}$$

The normalized two-point cross-correlation, R, is modeled as

$$\begin{aligned} R = \exp\left[-\frac{(\xi - \overline{u}\tau)^2}{l_{sx}^2}\right] \exp\left[-\frac{(1 - \tanh[\alpha|\xi|])|\xi - \overline{u}\tau|}{l_{sx}}\right] \\ \times \exp\left[-\frac{|\xi|}{l_{sx}}\right] \exp\left[-\frac{|\eta|}{l_{sy}}\right] \exp\left[-\frac{|\zeta|}{l_{sz}}\right], \end{aligned} \tag{12}$$

Acoustic Prediction Models

Time-domain model for ρ, $\boldsymbol{u}$, and p

$$q_k^{\perp}(\boldsymbol{x}, t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{n=0}^{4} \boldsymbol{q}_{g,k}^{\perp n}(\boldsymbol{x}, t; \boldsymbol{y}, \tau)\, \Theta_n(\boldsymbol{y}, \tau)\, d\tau\, d\boldsymbol{y} \tag{13}$$

and in the spectral domain for same field-variables

$$S_k^{\perp}(\boldsymbol{x}, \omega) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{m=0}^{4} \sum_{n=0}^{4} \hat{q}_{g,k}^{*\perp m}(\boldsymbol{x}; \boldsymbol{y}, \omega)\, \hat{q}_{g,k}^{\perp n}(\boldsymbol{x}; \boldsymbol{y} + \boldsymbol{\eta}, \omega) \times R_{m,n}^{\perp}(\boldsymbol{y}, \boldsymbol{\eta}, \tau)\, d\tau\, d\boldsymbol{\eta}\, d\boldsymbol{y}. \tag{14}$$

Exact Sources for Noise from Turbulence

Continuity

$$\Theta_0 = -\frac{\partial \underline{\rho}}{\partial t} - \frac{\partial \underline{\rho}\,\underline{u}_j}{\partial x_j}, \tag{15}$$

the momentum sources are

$$\Theta_i = -\frac{\partial}{\partial t}\left[\underline{\rho}\,\underline{u}_i\right] - \frac{\partial}{\partial x_j}\left[\underline{\rho}\,\underline{u}_i\underline{u}_j\right] + \frac{\partial}{\partial x_j}\left\{\mu\left[\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right]\right\} - \frac{2}{3}\frac{\partial}{\partial x_j}\left\{\mu\frac{\partial}{\partial x_k}\underline{u}_k\right\} - \frac{\partial \underline{p}}{\partial x_j}\delta_{ij}, \tag{16}$$

for $i = 1$ to 3, and the energy source is

$$\begin{aligned}\Theta_4 = &-\frac{\partial \underline{p}}{\partial t} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\,\underline{u}_k\underline{u}_k}{\partial t} - \gamma\frac{\partial \underline{u}_j\underline{p}}{\partial x_j} - \frac{\gamma-1}{2}\frac{\partial \underline{\rho}\,\underline{u}_j\underline{u}_k\underline{u}_k}{\partial x_j} + (\gamma-1)\frac{\partial}{\partial x_j}\left(\frac{c_p\mu}{\mathcal{P}r}\frac{\partial}{\partial x_j}\underline{T}\right)\\ &+(\gamma-1)\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\left(\frac{\partial \underline{u}_i}{\partial x_j} + \frac{\partial \underline{u}_j}{\partial x_i}\right)\right] - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x_j}\left[\mu\underline{u}_i\frac{\partial \underline{u}_k}{\partial x_k}\right].\end{aligned} \tag{17}$$

Noise Source Terms from Small-Scale Turbulence Written as Two-Point Correlation

Dominant sources of noise from small-scale turbulent structures within fully developed turbulence are written as two-point cross-correlations

$$R^{(8)} = \left\langle \frac{D\breve{p}^{(1)}}{Dt}, \frac{D\breve{p}^{(2)}}{Dt} \right\rangle \approx \frac{\overline{\rho}^2 K_s^2}{\tau_s^2} \breve{R} \approx \overline{\rho}^2 K_s^2 \Omega^2 \breve{R}, \tag{18a}$$

$$R^{(9)} = \left\langle \frac{D\breve{p}^{(1)}}{Dt}, \frac{D\overline{\rho}^{(2)} \breve{u}_k^{(2)} \breve{u}_k^{(2)}}{Dt} \right\rangle \approx \frac{\overline{\rho}^2 K_s^2}{\tau_s^2} \breve{R} \approx \overline{\rho}^2 K_s^2 \Omega^2 \breve{R}, \tag{18b}$$

and

$$R^{(10)} = \left\langle \frac{D\overline{\rho}^{(1)} \breve{u}_k^{(1)} \breve{u}_k^{(1)}}{Dt}, \frac{D\overline{\rho}^{(2)} \breve{u}_k^{(2)} \breve{u}_k^{(2)}}{Dt} \right\rangle \approx \frac{\overline{\rho}^2 K_s^2}{\tau_s^2} \breve{R} \approx \overline{\rho}^2 K_s^2 \Omega^2 \breve{R}. \tag{18c}$$

Noise Source Terms from Shockwave Shear Layer Interaction Written as Two-Point Correlation

A rare source term in the time-domain

$$\gamma \frac{\partial \hat{u}_j^{(1)} \overline{p}^{(1)}}{\partial y_j} \tag{19}$$

and spectral domain

$$\left\langle \gamma \frac{\partial \hat{u}_j^{(1)} \overline{p}^{(1)}}{\partial y_j}, \gamma \frac{\partial \hat{u}_m^{(2)} \overline{p}^{(2)}}{\partial y_m} \right\rangle \tag{20}$$

Approximations of Scaling Terms for Small Scale Turbulent Noise – Example Prediction

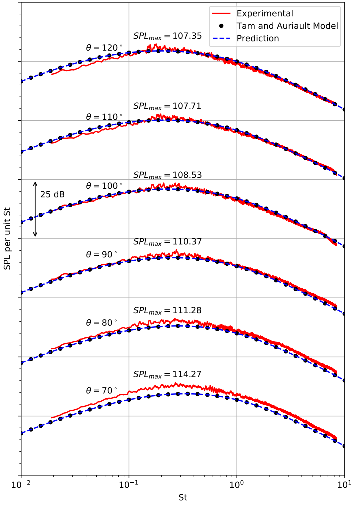

Figure 2: Prediction of fine-scale mixing noise at NPR = 1.893 and TTR = 3.2.

A More Advanced Spectral Approach

$$\widehat{\left\langle \tilde{\tilde{u}}_i \right\rangle}(\boldsymbol{y}, \tau) = \sqrt{\frac{2}{3}} \left(\int_{\kappa_1}^{\kappa_2} E_u(\boldsymbol{y}, \kappa, \tau) d\kappa \right)^{1/2}, \tag{21}$$

$$\widehat{\left\langle \tilde{\tilde{\rho}} \right\rangle}(\boldsymbol{y}, \tau) = \left(\int_{\kappa_1}^{\kappa_2} E_\rho(\boldsymbol{y}, \kappa, \tau) d\kappa \right)^{1/2}, \tag{22}$$

$$\widehat{\left\langle \tilde{\tilde{p}} \right\rangle}(\boldsymbol{y}, \tau) = \left(\int_{\kappa_1}^{\kappa_2} E_p(\boldsymbol{y}, \kappa, \tau) d\kappa \right)^{1/2}, \tag{23}$$

and,

$$\widehat{\left\langle \tilde{\tilde{T}} \right\rangle}(\boldsymbol{y}, \tau) = \left(\int_{\kappa_1}^{\kappa_2} E_T(\boldsymbol{y}, \kappa, \tau) d\kappa \right)^{1/2}, \tag{24}$$

where E_u, E_ρ, E_p, and E_T are the 'energy' spectra of the difference functions of the field variables, and κ_1 and κ_2 are the limits of integration.

Example Prediction from DNS Turbulence

Comparison of DNS with acoustic theory (no CFD) using wavenumber spectra from high-Re homogeneous isotropic turbulence

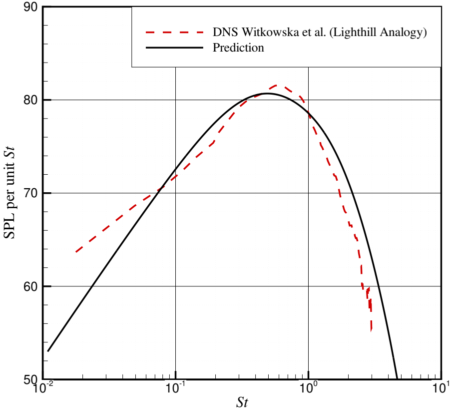

Figure 3: The power spectrum of acoustic pressure.

Noise Model for Large-Scale Coherent Structures

- Closed form analytical models for the noise from large-scale structures are absent and being sought after in the community.
- One such skeleton for the model might look like...

$$S(\boldsymbol{x},\omega) = \frac{\pi\omega^4}{c_\infty^4} g(\boldsymbol{x},\omega) g^*(\boldsymbol{x},\omega) \int_{-\infty}^{\infty} A_{ijlm} \frac{r_i r_j r_l r_m}{r^4} \frac{l_s l_{sy} l_{sz}}{\bar{u}} \exp\left[\frac{-l_s^2 \omega^2}{4\bar{u}^2}\right] \times \int_{-\infty}^{\infty} \exp\left[\frac{-i\xi\omega}{\bar{u}}\right] \exp\left[\frac{-|\xi|}{\bar{u}\tau_s}\right] d\xi \, dy_1 \quad (25)$$

Note appearance of typical RANS outcome and l_s. Model formulated this way on purpose to use RANS, intermitancy and transient information is missing. Is it impossible to create a large-scale model from steady RANS?

Contemporary Approaches
RANS Closures versus Aeroacoustic Sources

RANS

- Typically yields $\overline{q}$ and $k-\omega$ or ϵ
- Reynolds stresses are very helpful as have more direct connection with source models (e.g. Lighthill's model)

Aeroacoustics

- Typically requires estimation of two-point correlations through model involving l and τ
- Requires calibration of additional empirical coefficients beyond RANS closure

Desirable Characteristics
RANS Closures versus Aeroacoustic Sources

RANS

- Wavenumber spectra of field-variables
- Two-point correlations (is that possible?)
- Possible to form a RANS closure that yields aeroacoustic sources directly?

Aeroacoustics

- Formulation of models for various sources in time and two-point correlation
- Formulation of a closed-form large-scale model
- Seamless integration with turbulence models at the same level of LES/FWH approach.
- Eliminate or reduce empirical coefficients

Thank You