Why is a Supersonic Aircraft like a Tornado
or
Why is a Raven like a Writing Desk?

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Outline

- Brief biography and my research
- Introduction to jets and tornados
- Curious questions and a tangled tale
- Navier–Stokes equations
- An analogy and its solution
- Predictions for jets and tornados and their comparison
- Summary and conclusions

Brief Biography – Miller

- University of Florida (UF) Mechanical and Aerospace Engineering (MAE) professorship
  - US Air Force AFRL Faculty Fellow 2019
  - Previous life NASA
    - Civil Servant from 2009 - 2016
    - Research Aerospace Engineer – Theoretical Aeroacoustics
- Education
  - Ph.D. (NASA Grant) Aerospace Engineering, Penn State
  - M.S. (NREL Grant) Aerospace Engineering, Penn State
  - B.S. Mechanical Engineering, Michigan State University
  - Studies at Tagonrog State University (now Rostov), Russia
  - Eastern Michigan University
- Early life in Michigan

Theoretical Fluid Dynamics and Turbulence Group

- Interested in understanding turbulence physically and mathematically
- Interested in understanding how sound is produced by and propagated through turbulent fields
- Central questions within the field of fluid dynamics
- Multiple ways to solve problems - analytical, computationally, and experimentally
- My research focuses on analytical and computational (combined) methods

Figure 1: Leonardo da Vinci – notebook c. 1485.

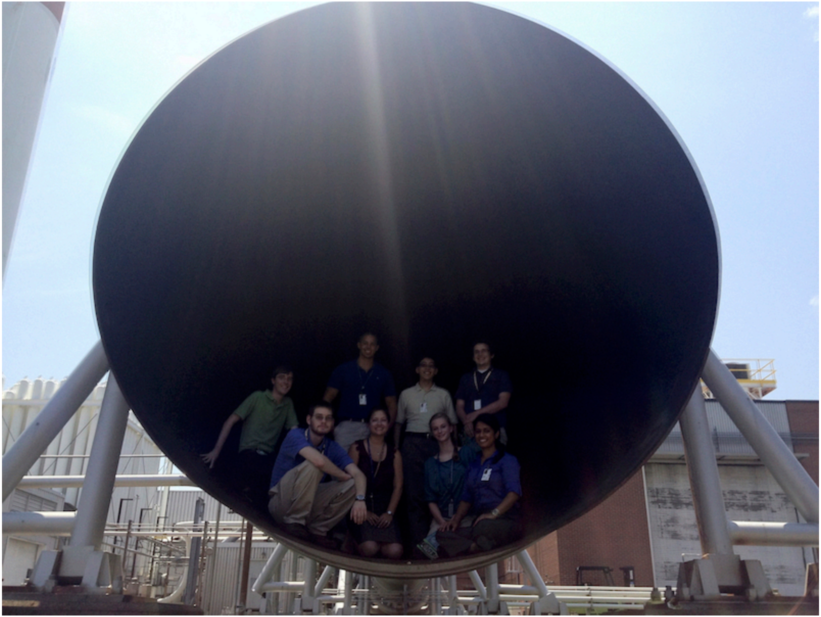


Figure 2: The student group of Prof. Miller at NASA Langley Research Center. Located at the exit of the 8-ft Hypersonic Wind Tunnel. c. 2015.

Contemporary Aircraft

Figure 3: Illustration of the X-59 QueSST landing on a runway. – LM illustration via NASA.gov

Figure 4: JSF take-off courtesy of U.S. ONR (see Martens 2018).

Physics of Turbulent Supersonic Jets

The jet flow-field is very complicated containing subsonic, transonic, and supersonic flows, shocks, expansions, turbulence, heating, and sometimes chemistry.

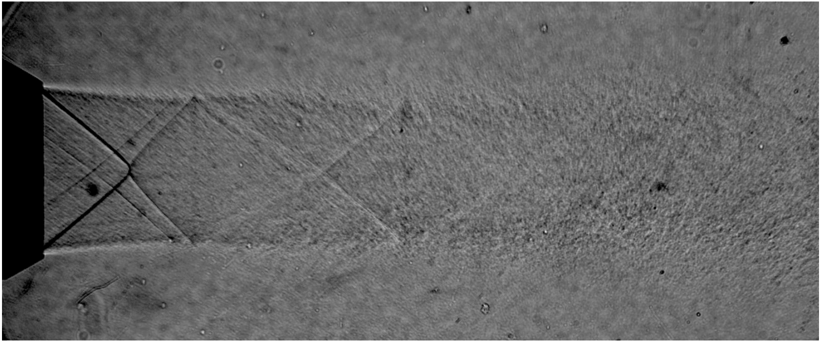


Figure 5: Schlieren of off-design supersonic jet courtesy of Zaman et al. 2011.

Tornado

Figure 6: Tornado damage in Joplin, Missouri, May 23, 2011, KOMUnews, Creative Commons.

Figure 7: Taken through Miller's NOAA 2018 program. Photograph of a TTU Ka-band radar and the striated updraft of a supercell near Imperial, KS on May 27th, 2019.

A Curious Question

- Why is a supersonic jet like a tornado?
- They both produce something beautiful ...
  - Let us go back in time ... to Victorian England

Figure 8: Alice's Adventure in Wonderland – Sir John Tenniel, 1864.

Excerpt from Alice in Wonderland – 1865

The Hatter opened his eyes very wide on hearing this; but all he said was "Why is a raven like a a writing-desk?"

"Come, we shall have some fun now!" thought Alice. "I'm glad they've begun asking riddles – I believe I can guess that," she added aloud.

"Do you mean that you think you can find out the answer to it?" said the March Hare. "Exactly so," said Alice. "Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least-at least I mean what I say - that's the same thing, you know." "Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!" "You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"

"You might just as well say," added the Dormouse, which seemed to be talking in its sleep, "that 'I breathe when I sleep' is the same thing as "sleep when I breathe'!"

"It is the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute, while Alice thought over all she could remember about ravens and writing-desks, which wasn't much.

Rev. Charles Dodgson (Lewis Carroll), 1865

Figure 9: Sir John Tenniel, 1864.

Ravens and Writing Desks

Figure 10: Edgar Allen Poe's "The Raven" by John Tenniel (1858). Note Sir J. Tenniel is the same illustrator for the Alice series!

Figure 11: Table of the court of King Louis XIV (Louis Dieudonné) c. 1670 – Smithsonian, Washington DC. (source: S. Miller).

Particular Answers to the Raven Riddle

In 1991 The Spectator, in England, asked for answers to the Hatter's riddle as its competition No. 1683. The winners, listed on July 6, are as follows:

- Because one is good for writing books and the other better for biting rooks. (George Simmers)
- Because a writing-desk is a rest for pens and a raven is a pest for wrens. (Tony Weston)
- Because "raven" contains five letters, which you might equally well expect to find in a writing desk. (Roger Baresel)
- Because they are both used to carrion decomposition. (Noel Petty)
- Because they both tend to present unkind bills. (M.R. Macintyre)
- Because they both have a flap in oak. (J. Tebbutt)
- Because it bodes ill for owed bills. Because they each contain a river–Neva and Esk.

Carroll's Reponse

"Enquiries have been so often addressed to me, as to whether any answer to the Hatter's Riddle can be imagined, that I may as well put on record here what seems to me to be a fairly appropriate answer, viz: 'Because it can produce a few notes, tho they are very flat; and it is never put with the wrong end in front!' This, however, is merely an afterthought; the Riddle, as originally invented, had no answer at all."

Rev. Charles Dodgson (Lewis Carroll) 1896

Figure 12: Auto–portrait of Rev. Charles Dodgson (Lewis Carroll) published 1896 – Public Domain.

Ravens and Writing Desks

- The same answer is true for jets and tornadoes! (but noisier)
- How are they related mathematically?
- How would we predict the noise from a jet? Or a tornado? The clue is that turbulence creates noise in both (including the raven!)
- We answer our question from the point of prediction – *the purpose of theory is prediction*.

Figure 13: Tenniel, 1864.

The Navier–Stokes Equations

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

Momentum equation

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_j}\delta_{ij} + \frac{\partial \tau_{ij}}{\partial x_j}$$

Energy equation

$$\frac{\partial \rho e_o}{\partial t} + \frac{\partial \rho u_j e_o}{\partial x_j} = -\frac{\partial u_j p}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \frac{\partial u_j \tau_{ij}}{\partial x_j}$$

where e_o is the total energy per unit mass $e_o = e + u_k u_k / 2$

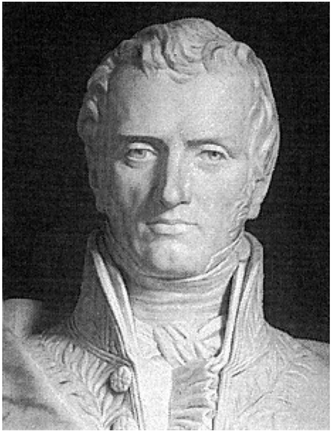
$$\tau_{ij} = 2\mu S_{ij} \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \qquad q_j = -c_p \frac{\mu}{\mathcal{P}_r}\frac{\partial T}{\partial x_j}$$

$$\mathcal{P}_r = c_p \mu \lambda^{-1}$$

Claude Louis Marie Henri Navier

10 February 1785 – 21 August 1836, French

- Engineer and physicist who specialized in mechanics
- Known for Navier-Stokes equations
- École polytechnique, and in 1804 continued his studies at the École Nationale des Ponts et Chaussées, from which he graduated in 1806
- Construction of bridges at Choisy, Asnières and Argenteuil
- Professor of calculus and mechanics at the École Polytechnique
- Named on Eiffel tower



George Gabriel Stokes

13 August 1819 – 1 February 1903, British

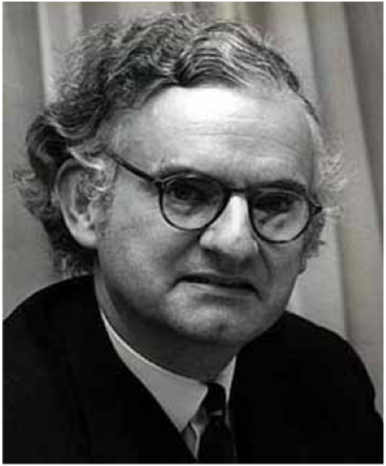
- Physicist and mathematician, fluids, optics
- Navier-Stokes equations
- Stokes spent all of his career at the University of Cambridge, where he served as Lucasian Professor of Mathematics
- President Royal Society
- President of the Victoria Institute, which had been founded to defend evangelical Christian principles against challenges from the new sciences, especially the Darwinian theory of biological evolution
- Married Mary Susanna Robinson, daughter of the Rev Thomas Romney Robinson, 5 children

It Gets Curiouser and Curiouser

- That is well and good that we have *a system of partial differential equations that model turbulence*
- Both the jet, tornado, and all other flow phenomena are governed by the same equations that model turbulence
  - But the *Navier–Stokes equations have no known analytical solution* for high-speed turbulent flows or tornadoes, or even the simplest of turbulent flows
  - Clay Mathematics Problem is trivial compared to the present problem, which deals with incompressible form for existence and uniqueness of solutions
- Let us predict the noise from a general turbulent flow then apply the solution to particular turbulent flows (jets and tornados)

Sir Michael James Lighthill
23 January 1924 – 17 July 1998, British

- Applied Mathematician
- Created acoustic analogy
- Worked on aeroacoustics, fluid dynamics, supersonic flight (Concorde), manned spacecraft
- Lucasian Professor of Mathematics, succeeded by Prof. Stephan Hawking
- Lighthill report led to AI Winter in the UK
- Open water swimmer



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Lighthill Acoustic Analogy

- Lighthill, in his now famous works on general theory (1952) and turbulence as a source of sound (1954), introduced the acoustic analogy.

Combining the time derivative of the continuity equation with the divergence of the momentum equation, an inhomogeneous wave equation results from the Navier-Stokes equations,

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 \mathcal{T}_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where $\mathcal{T}_{ij} = \rho u_i u_j - \tau_{ij} + (p - \rho c_\infty^2)\delta_{ij}$ is the Lighthill stress tensor.

- Lighthill constructed a model for $\mathcal{T}_{ij}$ consisting of convecting quadrupoles that resulted in an estimate of jet noise intensity scaling as u_j^8.
- That is really nice but we need to solve this equation ...

George Green
14 July 1793 – 31 May 1841, British

- British mathematical physicist
- Only received one year of formal schooling as a child– self-taught the rest
- Published *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism* in 1828
- In 1832 he was encouraged to receive formal education and was admitted to Gonville and Caius College
- After graduating was elected fellow of the Cambridge Philosophical Society and published six publications on hydrodynamics, sound, and optics
- Died in 1841 but four years later his work was discovered by Lord Kelvin who popularized it
- His work went on to be important in the development of quantum mechanics, electrodynamics, and superconductivity

Green's Function I

- To solve partial differential equations we can use the theory of vector Green's functions
- We illustrate the idea with a simple linear ODE

Consider the equation

$$T\frac{\partial^2 u}{\partial x^2} = f(x) \tag{2}$$

where T is a constant and $f(x)$ is a source function on x. Seek a solution for u by defining the Green's function from a source $\delta(x-\xi)$

$$T\frac{\partial^2 g}{\partial x^2} = \delta(x-\xi) \tag{3}$$

Recall the definition of the Dirac delta (δ) generalized function

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \tag{4}$$

Green's Function II

with

$$\delta(t) = \infty \text{ for } t = 0 \text{ and } \delta(t) = 0 \text{ for } t \neq 0 \tag{5}$$

We set boundary conditions for u and find subsequent boundary conditions for g (they are sometimes different). We solve for g

$$\begin{aligned} g(x;\xi) &= ax + b \text{ for } 0 \leq x < \xi \\ g(x;\xi) &= cx + d \text{ for } \xi < x \leq L \end{aligned} \tag{6}$$

We can now write the solution as

$$u(x) = \int_0^L f(\xi) g(x;\xi) d\xi \tag{7}$$

- For an introduction see Duffy, D. G. "Green's Functions with Applications," CRC Press, 2001.

Solution of Lighthill Acoustic Analogy

- Solution for ρ is found by the convolution integral of the Green's function of the wave equation with the double divergence of $\mathcal{T}_{ij}$.

We seek the Green's function of the wave equation

$$\frac{\partial^2 g}{\partial t^2} - c_\infty^2 \frac{\partial^2 g}{\partial x_i \partial x_i} = \delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau), \tag{8}$$

which is $g(x; y, t; \tau) = \frac{\delta(t-\tau-|\boldsymbol{x}-\boldsymbol{y}|/c_\infty)}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}$. The density field is

$$\rho - \rho_\infty = \frac{1}{4\pi c_\infty^2} \frac{\partial^2}{\partial x_i \partial x_j} \int\int\int \mathcal{T}_{ij}\left(\boldsymbol{y}, t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_\infty}\right) \frac{d\boldsymbol{y}}{|\boldsymbol{x}-\boldsymbol{y}|}. \tag{9}$$

Prediction Approach

- Ascertain boundary conditions, computational domain, observer locations
- Create computational fluid dynamic (CFD) solution via supercomputer OR model flow-field with statistical methods
- Integrate flow-field via acoustic analogy approach to find fluctuating values
- Find spectral density of acoustic pressure, p', via appropriate Fourier transforms ($S = \mathcal{F}(p')\mathcal{F}^*(p')$)
- Validate prediction approach with experimental measurement

Jet Predictions

- Careful experiments and computations yield insight into high speed turbulent jets and acoustic radiation.
- Numerical simulation takes 30 days of computer time on 2000 CPUs (2020).

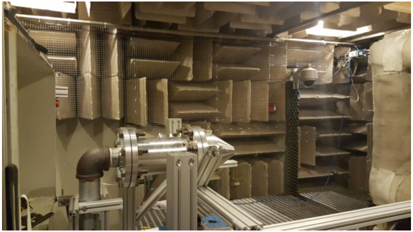

Figure 14: University of Florida Unsteady Fluid Dynamics Group Anechoic Chamber with installed nozzle for studying jet turbulence and acoustics.

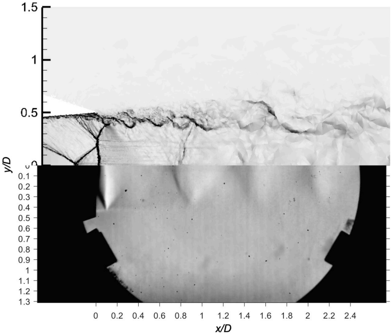

Figure 15: Example computational fluid dynamics comparison with experiment (schlieren, bottom).

Tornado Predictions

- Measurement of weather data and infrasound at tornado formation location for validation.
- Numerical simulations provided courtesy of Penn State and analyzed by Miller's group for sound source locations.

Figure 16: Infrasound microphone setup at tornado formation location on June 8th at c. 7:30 CDT.

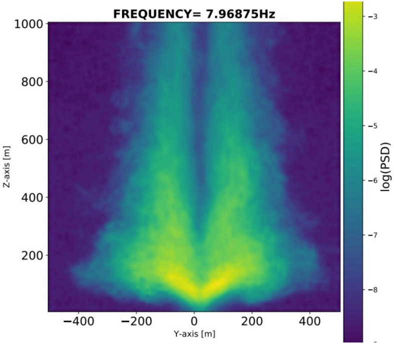

Figure 17: Contours of equivalent noise sources within EF5 tornado CFD simulation.

CFD and Noise from Jets and Tornados

Generally, *noise from turbulence is characterized as broadband and appears as "lobes" in the spectral domain.* The mathematical and physical reasons for this are another story.

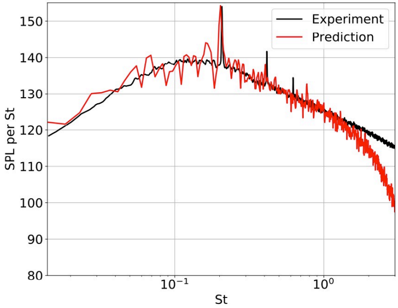

Figure 18: Predicted acoustic spectra from a jet flow.

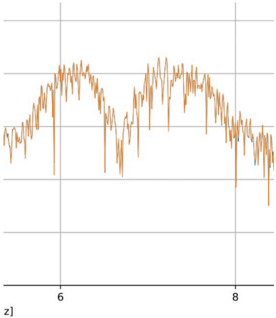

Figure 19: Predicted acoustic spectra from a tornado flow.

Summary and Conclusion

- Why is a raven like a writing desk?
  - *Carroll – Because they both can produce beautiful notes.*
- Why are jets like tornadoes?
  - *Miller – Because they both can produce beautiful noise.*
  - An improvement over Carroll's question relative to the present is that they are both based upon the *same equations*.
- Beautiful notes from a raven are governed by the same equations as those for the noise from jets and tornadoes – the Navier–Stokes equations

- The study of turbulence in engineering and math remains a topic that is heavily funded and even more mysterious
- Turbulence creates noise, and how this occurs is a physical and mathematical problem of great difficulty and relevance to society

Thank you ...

Questions?

Figure 20: Tenniel, 1864.