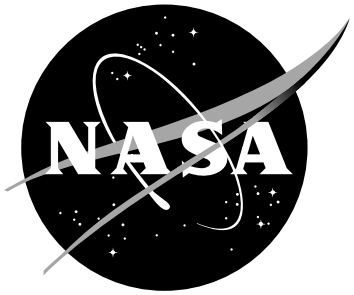
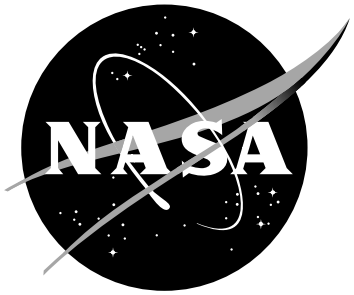


Toward Analytical Prediction of Acoustic Radiation from Isotropic Turbulence

Steven A. E. Miller
The National Aeronautics and Space Administration

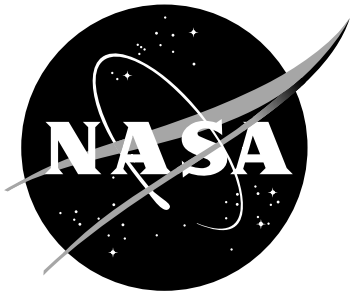
NASA Technical Working Group
October 20th – 21st 2015



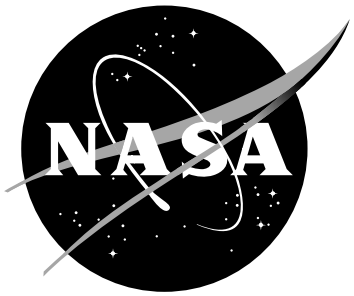
Acknowledgements

This work is possible because of the theories of
G. I. Taylor, A. A. Townsend, K. R. Sreenivasan,
A. N. Kolmogorov, G. K. Batchelor, S. Corrsin,
M. J. Lighthill, I. Proudman, and G. M. Lilley

Research conducted within the
National Aeronautics and Space Administration,
Advanced Air Vehicles Program,
Commercial Supersonic Technology Project



Introduction



Opportunity, Solution, and Benefit

Opportunity

- Almost all turbulent flows possess local isotropy
- Turbulence returns to isotropy
- Obtain more general and accurate predictions via decomposition containing isotropy

Solution

- Unique decomposition of Navier-Stokes equations
- Relate statistics to wavenumber spectra (exact statistical result)

Benefit

- Accurate prediction for isotropic portion of turbulent field
- Impactful theory that explains physics
- Potential to include anisotropic fluctuations

Isotropic Turbulence

Geoffrey Ingram Taylor
1886 - 1975

'in isotropic turbulence the average value of any function of the velocity components, defined in relation to a given set of axes, is unaltered if the axes of reference are rotated in any matter'

Taylor, G. I., "Statistical Theory of Turbulence," Proc. R. Soc. Lond. A, Vol. 151, No. 873, 1935, pp. 421–444. doi:10.1098/rspa.1935.0158.

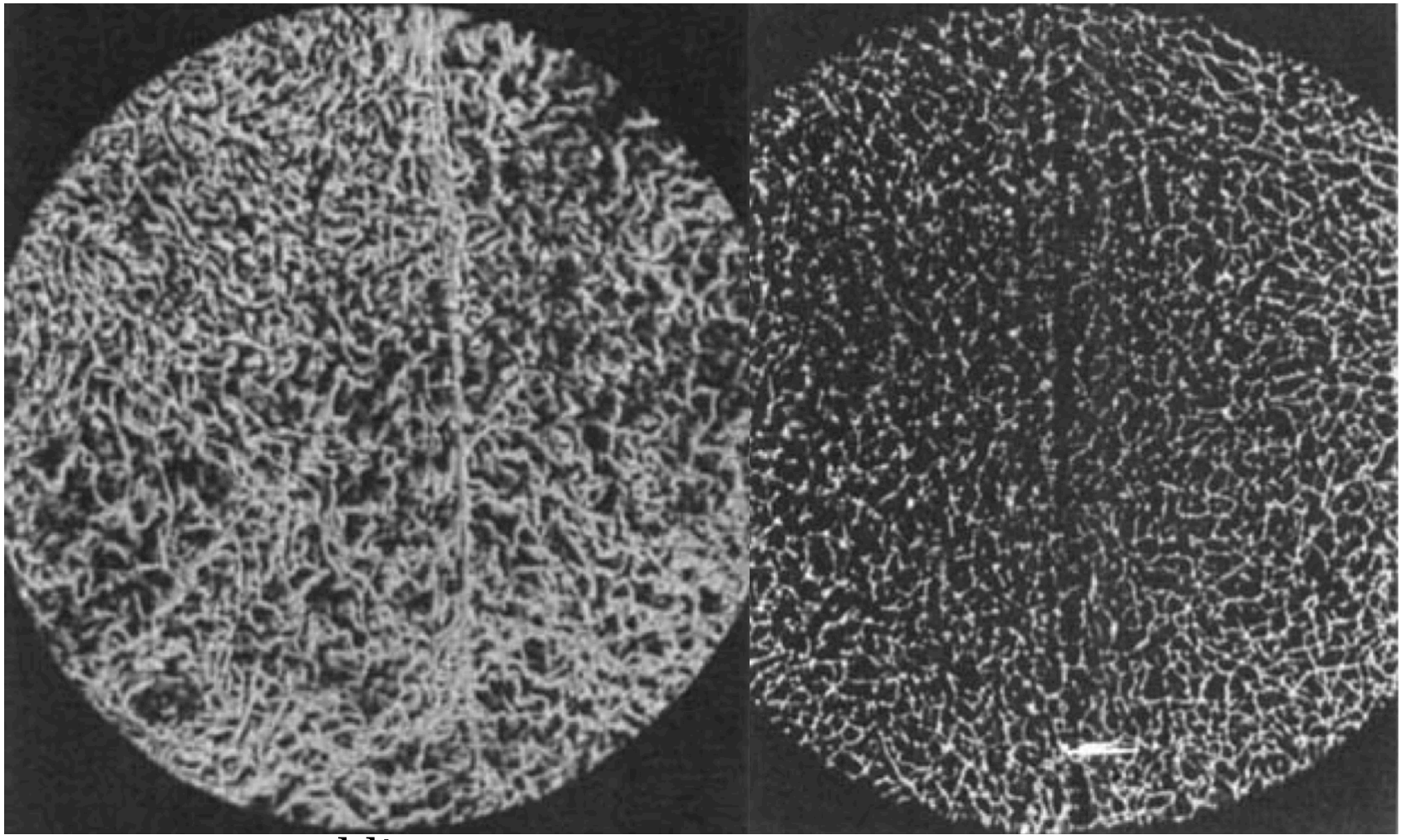
'*return to isotropy*'

Kolmogorov, A. N., "On Degeneration of Isotropic Turbulence in an Incompressible Viscous Liquid," Dokl. Akad. Nauk. SSSR, Vol. 31, 1941, pp. 147.

Andrei Kolmogorov
1903 - 1987

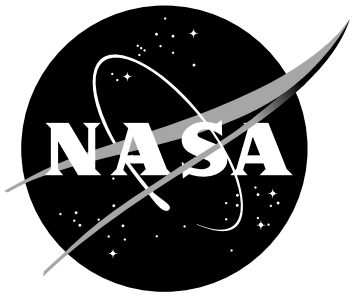
Stationary and decaying isotropic turbulence possesses similarity

Isotropic Turbulence



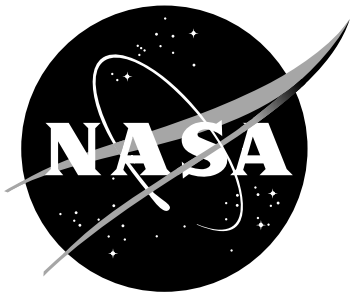
schlieren shadowgraph

Hesselink, L. and Sturtevant, B., "Propagation of Weak Shocks Through a Random Medium," Journal of Fluid Mechanics, Vol. 196, 1988, pp. 513–553. doi:10.1017/s0022112088002800.



Observations of Isotropic Turbulence

- Isotropic turbulence is observed
  - Grid turbulence (behind grids in wind tunnels, fences, etc.), where usually studied experimentally
  - Far from the wall in turbulent boundary layers
  - Atmosphere far from the ground
  - Within bounded flows such as pipes
  - Large spatial regions of jet flow
  - Large spatial regions of wakes
  - Local spatial regions of almost all flows
- Isotropy is not limited to traditional fluid dynamics
  - Magnetohydrodynamic flows
  - Plasmas in turbulent decay
  - Regions of interstellar medium
  - and more!



Universality: Wavenumber Spectra of Structure Functions

Velocity $$E_u \propto c_u \epsilon^{\alpha_u} \kappa^{-5/3}$$

Kolmogorov, A. N., "The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Number," Dokl. Akad. Nauk. SSSR, Vol. 30, 1941, pp. 299–303.

Pressure $$E_p \propto c_p \epsilon^{\alpha_p} \kappa^{-7/3}$$

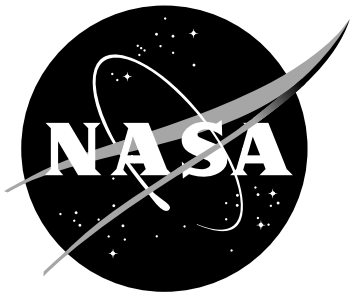
Batchelor, G. K., "Pressure Fluctuations in Isotropic Turbulence," Math. Proc. Camb. Phil. Soc., Vol. 47, No. 2, 1951, pp. 359–374. doi:10.1017/S030500410026712.

Temperature $$E_T \propto c_T \epsilon^{\alpha_T} \kappa^{-5/3}$$

Corrsin, S., "On the Spectrum of Isotropic Temperature Fluctuations in an Isotropic Turbulence," Journal of Applied Physics, Vol. 22, No. 4, 1951, pp. 469–473. doi: 10.1063/1.1699986.

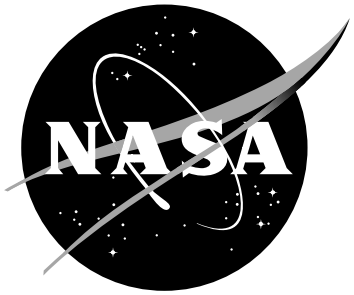
Density $$E_\rho \propto c_\rho \epsilon^{\alpha_\rho} \kappa^{-5/3}$$

Montgomery, D., Brown, M. R., and Matthaeus, W. H., "Density Fluctuation Spectra in Magnetohydrodynamic Turbulence," Journal of Geophysical Research, Vol. 92, No. A1, 1987, pp. 282–284. doi:10.1029/JA092iA01p00282.

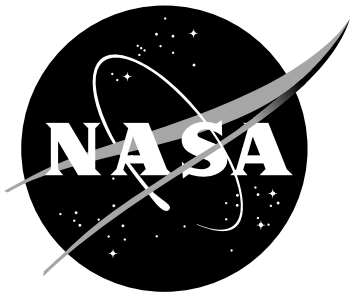


Notable Aeroacoustics Studies of Isotropic Turbulence

- Theoretical
  - Proudman, I., "The Generation of Noise by Isotropic Turbulence," The Royal Society: Proceedings A, Vol. 214, No. 1116, 1952, pp. 119–132. doi:10.1098/rspa.1952.0154.
  - Lilley, G. M., "The Radiated Noise from Isotropic Turbulence Revisited," NASA CR-191547, 1993.
  - Lilley, G. M., "The Radiated Noise from Isotropic Turbulence with Applications to the Theory of Jet Noise," Journal of Sound and Vibration, Vol. 190, No. 3, 1996, pp. 463–476. doi:10.1006/jsvi.1996.0074.
  - Lilley, G. M., "The Acoustic Spectrum in the Sound Field of Isotropic Turbulence," International Journal of Aeroacoustics, Vol. 4, No. 1, 2005, pp. 11–20. doi:10.1260/1475472053730011.
- Numerical
  - Sarkar, S. and Hussaini, M. Y., "Computation of the Sound Generated by Isotropic Turbulence," NASA-CR-191543, 1993, pp. 1–30.
  - Witkowska, A., Juve, D., and Brasseur, J. G., "Numerical Study of Noise from Isotropic Turbulence," Journal of Computational Acoustics, Vol. 5, No. 3, 1997, pp. 317–336. doi:10.1142/S0218396X97000186.
  - Seror, C., Sagaut, P., Bailly, C., and Juve, D., "On the Radiated Noise Computed by Large-Eddy Simulation," Physics of Fluids, Vol. 13, No. 476, 2001, pp. 476–487. doi:10.1063/1.1336150.
- Note: Many investigations of isotropic turbulence fluid dynamics



Mathematical Theory



Mathematical Theory Overview

- Decompose Navier-Stokes equations into time evolving base flow, anisotropic and isotropic components, and radiating waves
  - Source: Navier-Stokes equations operator of base flow and aerodynamic fluctuations
  - Radiation: Linearized Navier-Stokes equations operator on radiating waves
- Solution proposed in the time and spectral domain with unknown arguments
- Relate vector equivalent source to wavenumber spectra of decaying and stationary isotropic turbulence

Decomposition of Governing Equations

Decompose Navier-Stokes equations where q is a field-variable

$$\boldsymbol{q} = \overline{q} + \tilde{q} + \tilde{\tilde{q}} + q' + q''$$

and,

$\overline{q} =$ Time-evolving or steady non-radiating base flow

$\tilde{q} =$ Anisotropic turbulent fluctuations

$\tilde{\tilde{q}} =$ Isotropic turbulent fluctuations

$q' =$ Radiating waves associated with anisotropic turbulence

$q'' =$ Radiating waves associated with isotropic turbulence

Θ_i Sources are ***exactly*** Navier-Stokes equations operators on $\underline{\boldsymbol{q}}$

Solution through Method of Vector Green's Function

Solution has the form (but more unknowns than equations)

$$q_k^{\perp}(\boldsymbol{x},t)=\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}\sum_{n=0}^{4}q_{g,k}^{\perp n}(\boldsymbol{x},t;\boldsymbol{y},\tau)\,\Theta_n(\boldsymbol{y},\tau)\,d\tau d\boldsymbol{y}$$

where $q_k^{\perp}$ is the summation of radiation from isotropic and anisotropic turbulence.

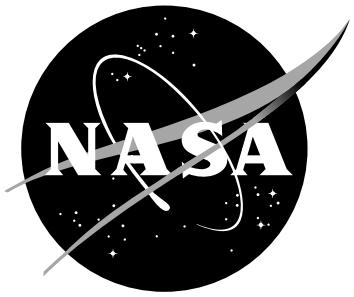
Θ_i is known and we need to find

$$\boldsymbol{q}_{g,k}^{\perp,n}=\left[\rho_{g,k}^{\perp,n},\boldsymbol{u}_{g,k}^{\perp,n},p_{g,k}^{\perp,n}\right]^T$$

We can also write the k^{th} spectral density

$$S_k^{\perp}(\boldsymbol{x},\omega)=\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}\sum_{m=0}^{4}\sum_{n=0}^{4}\hat{q}_{g,k}^{*\perp m}(\boldsymbol{x};\boldsymbol{y},\omega)\,\hat{q}_{g,k}^{\perp n}(\boldsymbol{x};\boldsymbol{y}+\boldsymbol{\eta},\omega)$$
$$\times\int_{-\infty}^{\infty}\Theta_m(\boldsymbol{y},\tau)\,\Theta_n(\boldsymbol{y}+\boldsymbol{\eta},\tau+\Delta\tau)\,d\Delta\tau d\tau d\boldsymbol{\eta}d\boldsymbol{y}$$

for $k = 0$ to 4.



Source Statistics

Each two-point cross-correlation of Θ_i is written analytically.
Example: continuity–energy ($k_1 = 0$ and $k_2 = 4$)

$$
\begin{aligned}
R^{\perp}_{0,4}\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right) = &\left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{p}^{(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\gamma-1}{2}\frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)}\underline{u}_n^{2,(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \gamma\frac{\partial \underline{u}_m^{(2)}\underline{p}^{(2)}}{\partial y_m} \right\rangle \\
&+ \left\langle \frac{\gamma-1}{2}\frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial \underline{\rho}^{(2)}\underline{u}_m^{(2)}\underline{u}_n^{(2)}\underline{u}_n^{(2)}}{\partial y_m} \right\rangle + \left\langle -\left(\gamma-1\right)\frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial}{\partial y_m}\left(\frac{c_p\mu}{Pr}\frac{\partial}{\partial y_m}\left[\underline{T}^{(2)}\right]\right) \right\rangle \\
&+ \left\langle -\left(\gamma-1\right)\frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \frac{\partial}{\partial y_m}\left[\mu\underline{u}_l^{(2)}\left(\frac{\partial \underline{u}_l^{(2)}}{\partial y_m}+\frac{\partial \underline{u}_m^{(2)}}{\partial y_l}\right)\right] \right\rangle + \left\langle \frac{2}{3}\frac{\partial \underline{\rho}^{(1)}}{\partial \tau}, \delta_{lm}\frac{\partial}{\partial y_m}\left[\mu\underline{u}_l^{(2)}\frac{\partial \underline{u}_n^{(2)}}{\partial y_n}\right] \right\rangle \\
&+ \left\langle \frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{p}^{(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\gamma-1}{2}\frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)}\underline{u}_n^{(2)}\underline{u}_n^{(2)}}{\partial \tau} \right\rangle + \left\langle \frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \gamma\frac{\partial \underline{u}_m^{(2)}\underline{p}^{(2)}}{\partial y_m} \right\rangle \\
&+ \left\langle \frac{\gamma-1}{2}\frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial \underline{\rho}^{(2)}\underline{u}_m^{(2)}\underline{u}_n^{(2)}\underline{u}_n^{(2)}}{\partial y_m} \right\rangle + \left\langle -\left(\gamma-1\right)\frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial}{\partial y_m}\left(\frac{c_p\mu}{Pr}\frac{\partial}{\partial y_m}\left[\underline{T}^{(2)}\right]\right) \right\rangle \\
&+ \left\langle -\left(\gamma-1\right)\frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \frac{\partial}{\partial y_m}\left[\mu\underline{u}_l\left(\frac{\partial \underline{u}_l^{(2)}}{\partial y_m}+\frac{\partial \underline{u}_m^{(2)}}{\partial y_l}\right)\right] \right\rangle \\
&+ \left\langle \frac{2}{3}\frac{\partial \underline{\rho}^{(1)}\underline{u}_j^{(1)}}{\partial y_j}, \delta_{lm}\frac{\partial}{\partial y_m}\left[\mu\underline{u}_l^{(2)}\frac{\partial \underline{u}_n^{(2)}}{\partial y_n}\right] \right\rangle
\end{aligned}
$$

Twenty-four more are derived and this one has relatively few terms.

Application of Theory of Isotropic Turbulence

Using the theory of isotropic turbulence we can simplify the continuity-energy relation (among others) as

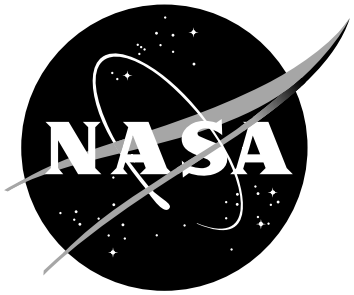
$$R_{0,4}^{\perp}\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right)\approx\left\{\left(-\frac{3\left(\overline{\rho}+\widehat{\left\langle\tilde{\tilde{\rho}}\right\rangle}\right)\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}}{l}-\frac{\left(\overline{\rho}+\widehat{\left\langle\tilde{\tilde{\rho}}\right\rangle}\right)}{\tau}\right)\left(-\frac{3\gamma\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}\left(\overline{p}+\widehat{\left\langle\tilde{\tilde{p}}\right\rangle}\right)}{l}-\frac{6\overline{\mu}\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}^{2}}{l^{2}}+\frac{3c_{p}\widehat{\left\langle\tilde{\tilde{T}}\right\rangle}\left(\gamma-1\right)\overline{\mu}}{\mathcal{P}rl^{2}}\right.$$
$$\left.+\frac{9\overline{\mu}\left(\gamma-1\right)\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}^{2}}{l^{2}}-\frac{9\left(\gamma-1\right)\left(\overline{\rho}+\widehat{\left\langle\tilde{\tilde{\rho}}\right\rangle}\right)\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}^{3}}{2l}-\frac{\widehat{\left\langle\tilde{\tilde{p}}\right\rangle}}{\tau}-\frac{3\left(\gamma-1\right)\left(\overline{\rho}+\widehat{\left\langle\tilde{\tilde{\rho}}\right\rangle}\right)\widehat{\left\langle\tilde{\tilde{u}}\right\rangle}^{2}}{2\tau}\right)\right\}\boldsymbol{R}$$

where *R* is the normalized two-point cross-correlation (not shown, but can easily use analytic *f(r) and g(r)*) and (for example)

$$\widehat{\left\langle\tilde{\tilde{u}}_{i}\right\rangle}\left(\boldsymbol{y},\tau\right)=\sqrt{\frac{2}{3}}\left(\int_{\kappa_{1}}^{\kappa_{2}}E_{u}(\boldsymbol{y},\kappa,\tau)d\kappa\right)^{1/2}$$

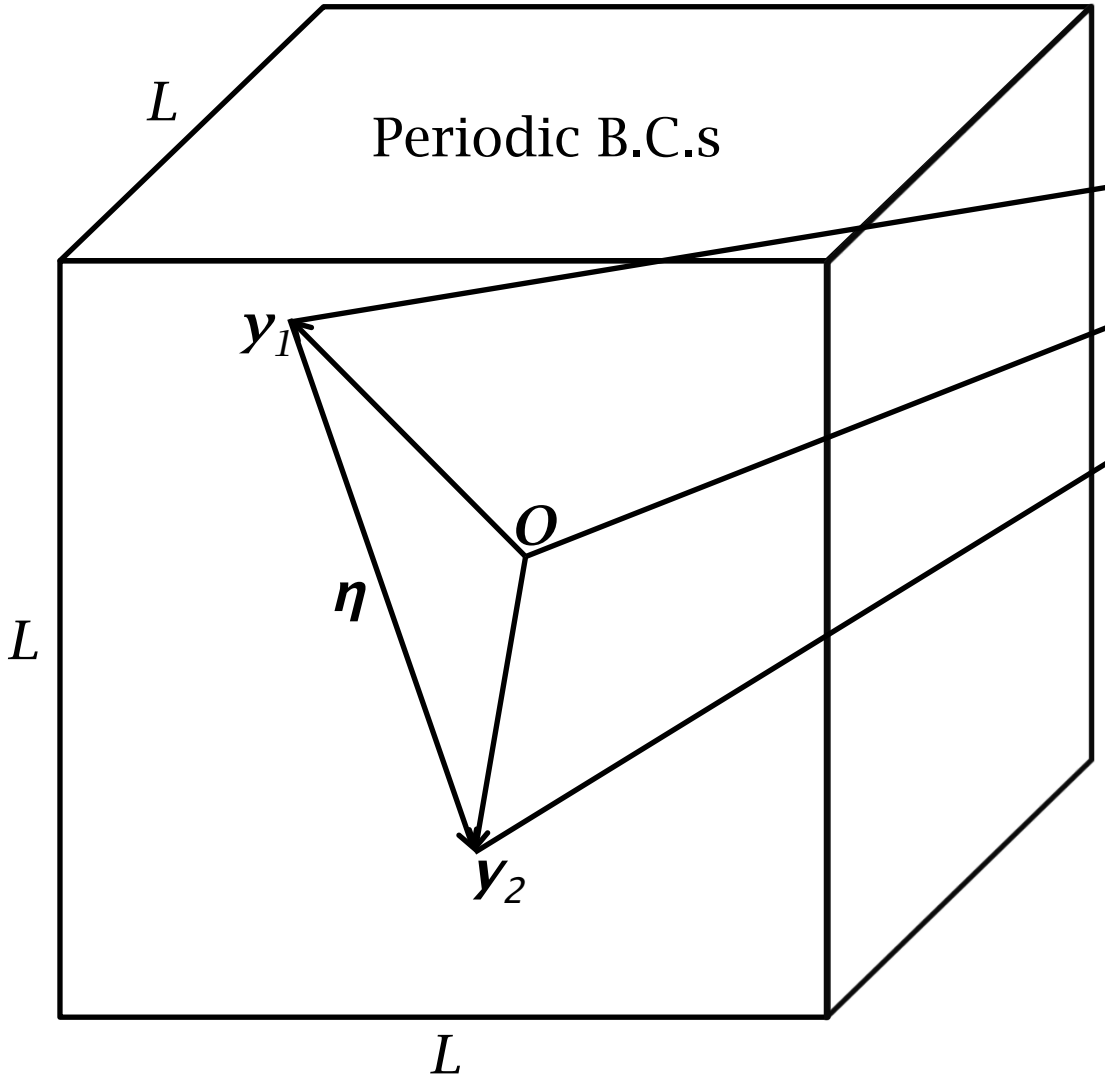
and E_i are the composite wavenumber spectra (for example)

$$E_{u}(\kappa)=c_{u}\epsilon^{2/3}\kappa^{-5/3}f_{L}\left(\kappa L\right)f_{\eta}\left(\kappa\eta\right)$$



Results

Coordinate System and Traditional Problem

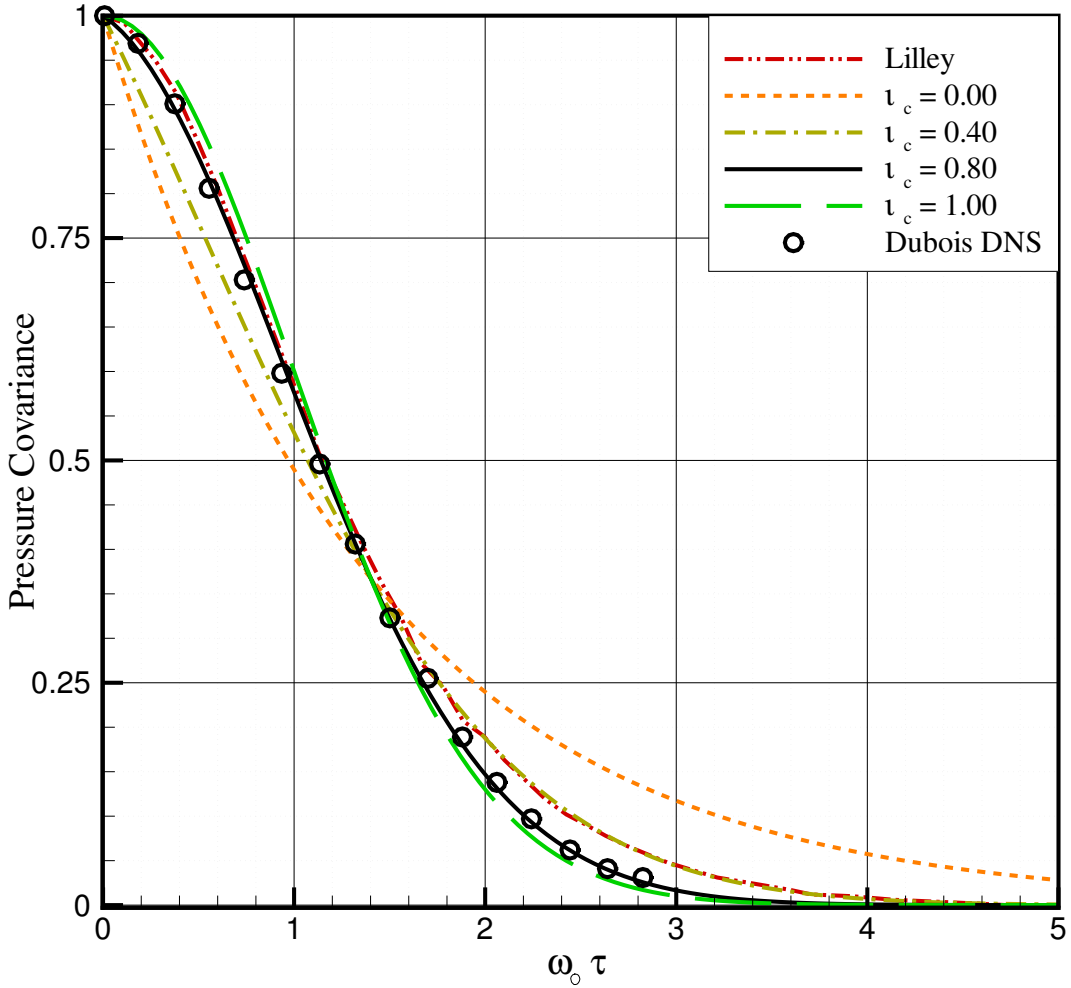


Coordinate System Nomenclature

- ***x*** Observer vector
- ***y*** Source vector
- ***η*** Source separation vector
- ***O*** Origin
- ***L*** Length of domain

Correlation of Pressure

- Two-point hybrid Gaussian-exponential model aligns with DNS
- $\mathcal{R}e_L = 6328$
- DNS of Debussche et al
  - 512^3 grid points

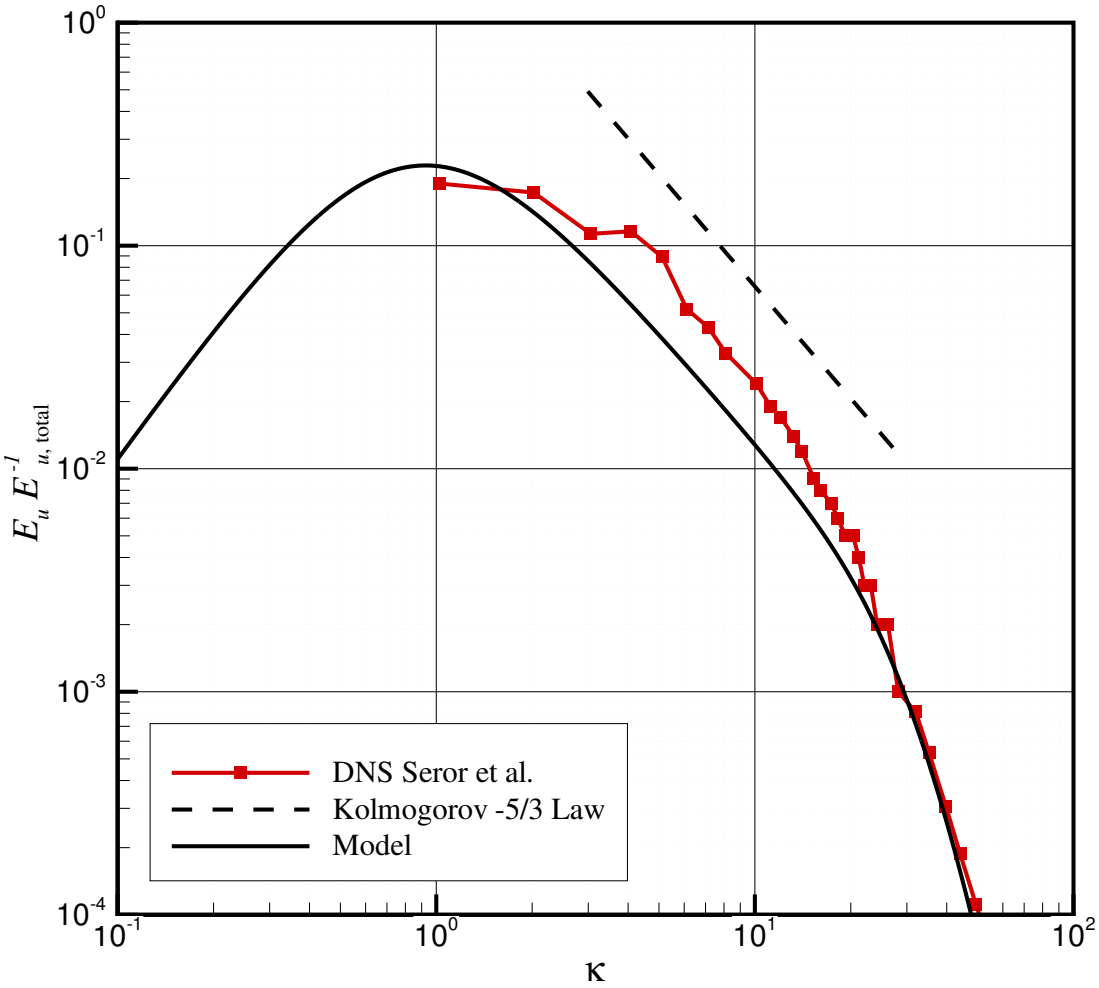


Lilley, G. M., "The Acoustic Spectrum in the Sound Field of Isotropic Turbulence," International Journal of Aeroacoustics, Vol. 4, No. 1, 2005, pp. 11–20. doi: 10.1260/1475472053730011.

Debussche, A., Dubois, T., and Temam, R., "The Nonlinear Galerkin Method: A Multi-Scale Method Applied to the Simulation of Homogeneous Turbulent Flows," ICASE Report 93/93, 1993.

Wavenumber Energy Spectrum E_u

- Composite wavenumber spectrum aligns with DNS and K41 theory
- $\mathcal{R}e_\lambda = 82.31$
- $\mathcal{L} = 0.98$ m
- $\lambda = 0.28$ m
- $\eta = 0.02$ m
- Seror et al DNS
  - Grid points 192^3



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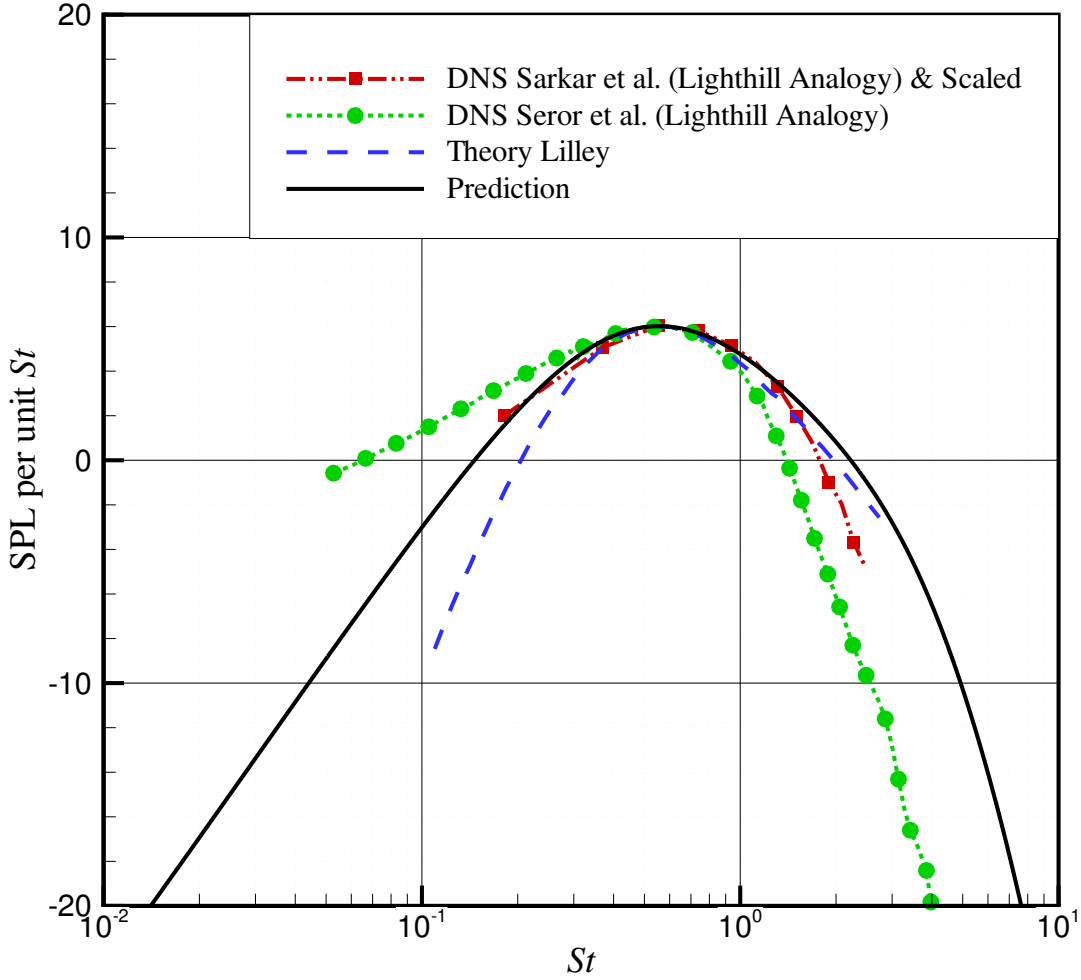
Spectrum of Acoustic Pressure

- Differential volume element
- Theory at high $\mathcal{R}e$, simulation at low $\mathcal{R}e$
- Stationary turbulence
- $\mathcal{R}e_\lambda = 50$
- Note $St = f\,\mathcal{L}\,k^{-1/2}$



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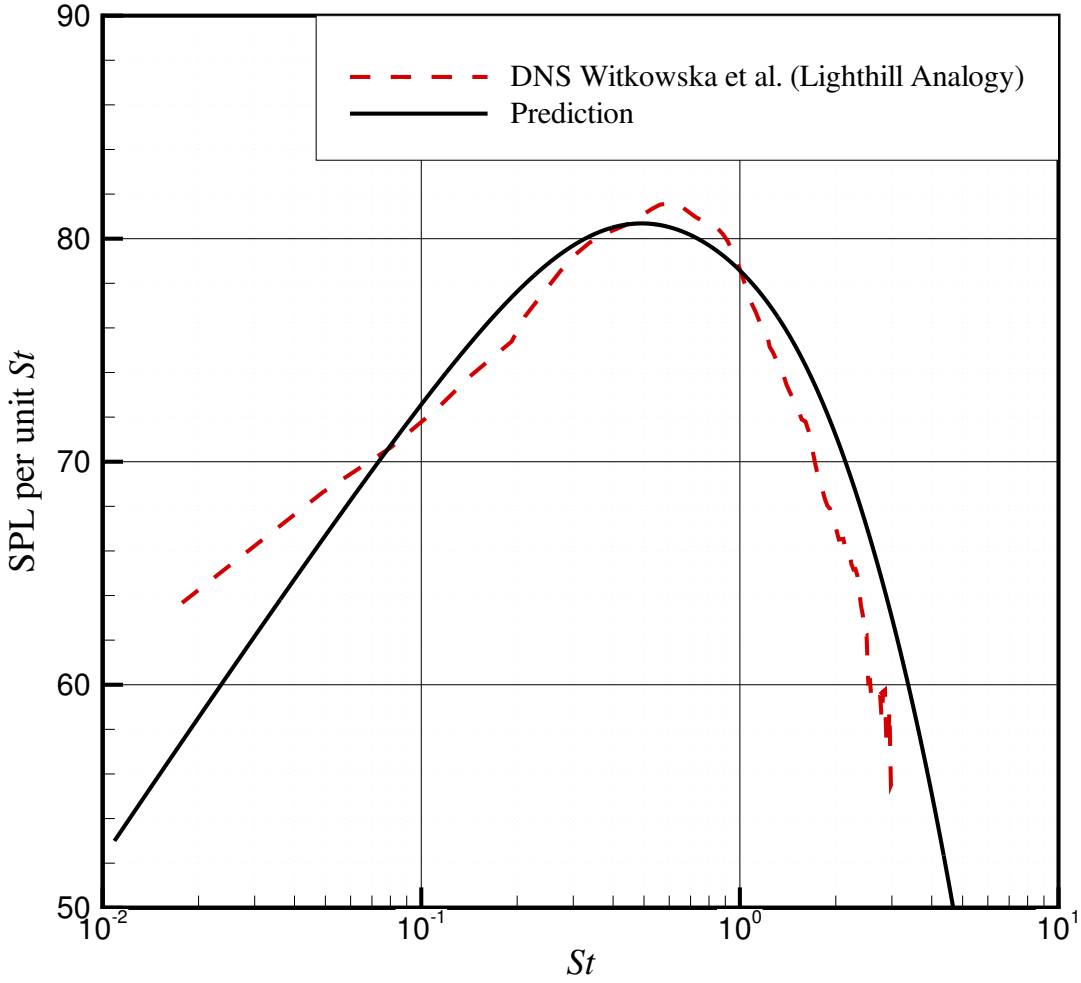
Spectrum of Acoustic Pressure

- Theory at high $\mathcal{R}e$ and applied at low $\mathcal{R}e$
- Simulations at low $\mathcal{R}e$
- Stationary turbulence
- $\mathcal{R}e_\lambda = 20$
- Domain $\approx 14.3\mathcal{L}$
- Observer at $286\mathcal{L}$
- Witkowska et al DNS
  - Grid points 128^3



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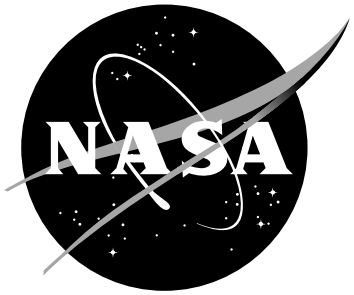
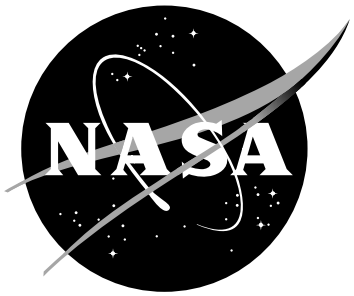
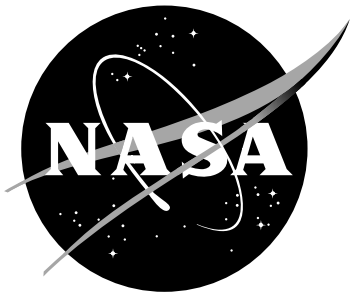


Research Summary and Conclusions



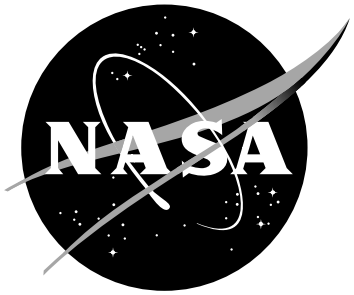
Immediate Research Goals

- Demonstrate theory is consistent with power law scaling of Proudman and Lilley
- Validate predictions of decaying k, $\langle TT \rangle$, $\langle pp \rangle$, ε, E_u, E_T, and E_p compared with DNS
- Validate normalized two-point cross-correlation as a function of time and wavenumber
- Predictions of acoustic power with increasing $\mathcal{R}e_{L,}$ k, etc.
- Additional acoustic pressure spectral comparisons
- Theoretical development of energy spectra of structure functions
- Anisotropic theory



Summary and Conclusion

- Isotropic Turbulence
  - *Return to isotropy*
  - Occurs universally
- Mathematical Theory
  - Unique decomposition of Navier-Stokes equations
  - Related sources to universal isotropic energy spectrum
- Predictions
  - Correlations, wavenumber spectrum, and acoustics agree with measurement and numerical experiment
- Research Ongoing
  - Full development shown in upcoming AIAA Paper
  - New theory being refined for decaying turbulence
- This work is a component of more general theory



Questions