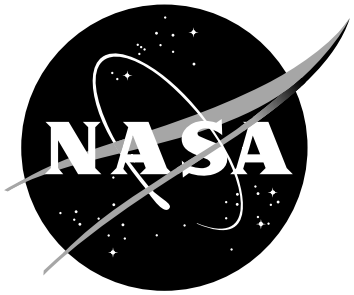
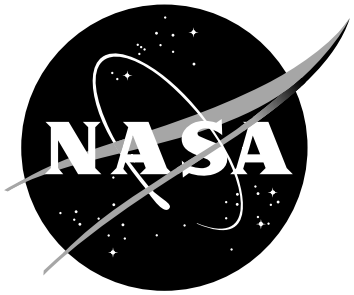


Prediction of Turbulent Boundary Layer Noise and Contemporary Challenges

Steven A. E. Miller
The National Aeronautics and Space Administration

NASA Technical Working Group
April 19th – 20th 2016

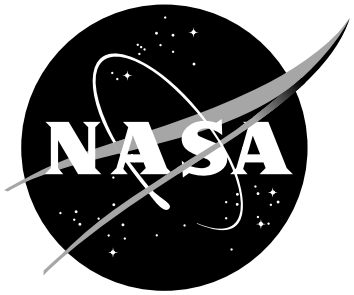


Acknowledgements

This work is possible because of
substantial contributions from
Prandtl, Kovasnay, Lighthill, Powell, and others

Research conducted within the
National Aeronautics and Space Administration,
Advanced Air Vehicles Program,
Commercial Supersonic Technology Project

This work is the third of a three part article series
Currently under journal review



Outline

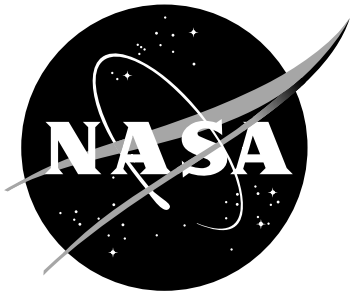
Introduction
- Problem overview
- Survey of previous approaches

Mathematical Theory
- Mathematical modeling goals
- Cross-spectral acoustic analogy
- Aerodynamic and acoustic models for boundary layers

Results
- Auto-spectral predictions

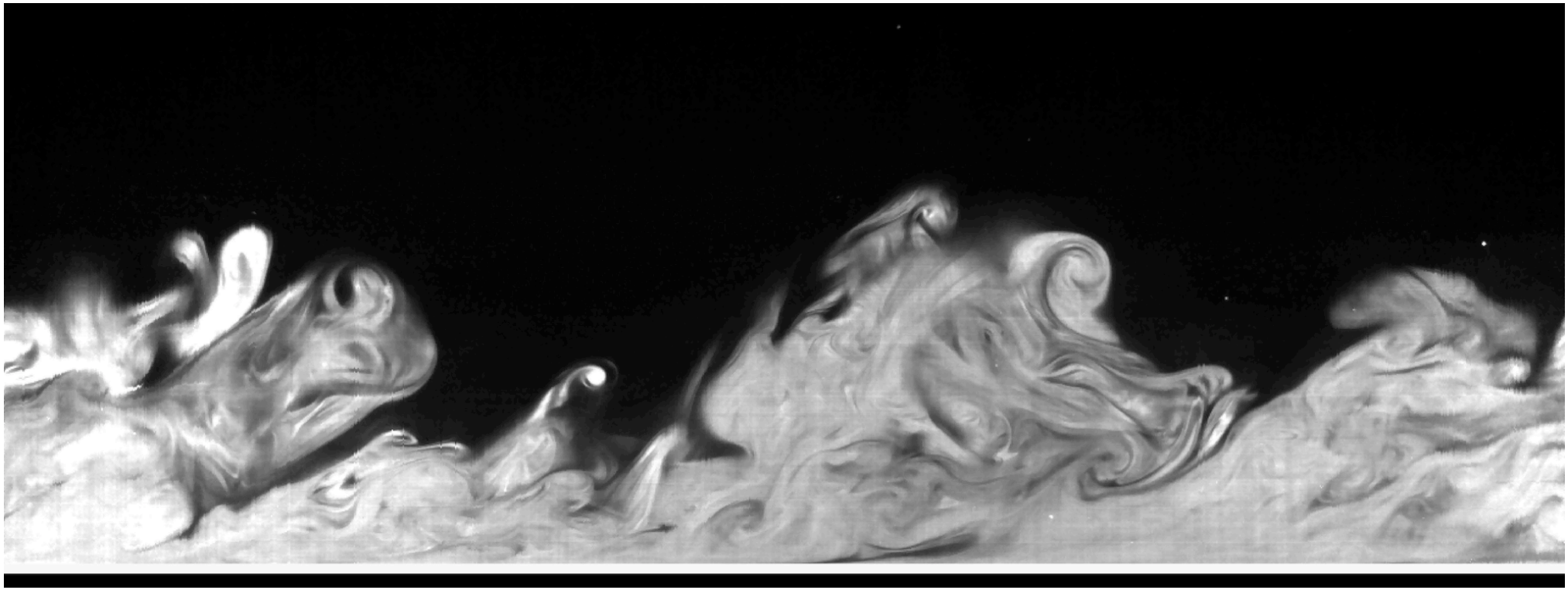
Contemporary Challenges
- Pressure gradient, roughness, permeability

Summary and Conclusion



Introduction

The Turbulent Boundary Layer Visualized



Lee, J. H., Kwon, Y. S., Monty, J. P., and Hutchins, N., "Tow-Tank Investigation of the Developing Zero-Pressure-Gradient Turbulent Boundary Layer," 18th Australasian Fluid Mechanics Conference, 2012.

Survey of Boundary Layer Turbulence Experimental Aeroacoustics

Hydrophone measurements in water tunnel

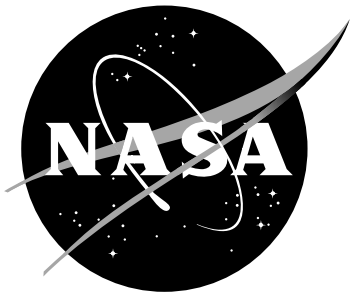
Greshilov, E. M. and Mironov, M. A., "Experimental Evaluation of Sound Generated by Turbulent Flow in a Hydrodynamic Duct," Soviet Physics - Acoustics, Vol. 29, No. 4, 1983, pp. 275–280.

Boundary layer noise in open and closed-wall aeroacoustics wind tunnels – shown to contribute below 500 Hz

Duell, E., Walter, J., Arnette, S., and Yen, J., "Boundary Layer Noise in Aeroacoustic Wind Tunnels," AIAA Paper 2004-1028, 2004. DOI:10.2514/6.2004-1028.

Variation of surface roughness to examine roughness effects

Smith, B., Alexander, W., Devenport, W., Glegg, S., and Grissom, D., "The Relationship Between Roughness Noise and the Near-Field Pressure Spectrum," AIAA Paper 2008-2904, 2008. DOI:10.2514/6.2008-2904.



Survey of Boundary Layer Turbulence Aeroacoustic Predictions

Likely first model for prediction of turbulent boundary layer noise

Powell, A., "Aerodynamic Noise and the Plane Boundary," Journal of the Acoustical Society of America, Vol. 32, No. 8, 1960, pp. 982–990. doi:10.1121/1.1908347.

A number of theoretical investigations without any validation

Ffowcs Williams, J. E., "Sound Radiation from Turbulent Boundary Layers Formed on Compliant Surfaces," Journal of Fluid Mechanics, Vol. 22, No. 2, 1965, pp. 347–358. doi: 10.1017/s0022112065000794. (Also DOI: 10.1017/s0022112072001338 and 10.1017/s0022112081003455)

Related wavenumber pressure spectrum on wall to noise

Howe, M. S., "Surface Pressures and Sound Produced by Turbulent Flow Over Smooth and Rough Walls," Journal of the Acoustical Society of America, Vol. 90, No. 2, 1991, pp. 1041–1047. doi:10.1121/1.402292.

Wall roughness models and measurements

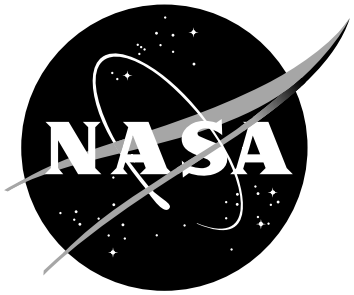
Glegg, S., Devenport, W., Grissom, D., and Smith, B., "Rough Wall Boundary Layer Noise: Theoretical Predictions," AIAA Paper 2007-3417, 2004. doi:10.2514/6.2007-3417.

DNS combined with an acoustic analogy and a half-space Green's function

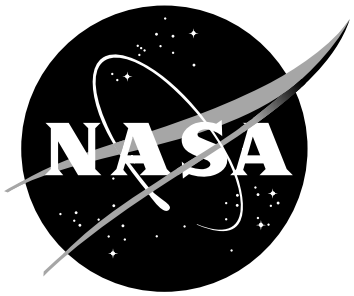
Hu, Z.,Morfey, C., and Sandham, N. D., "Sound Radiation in Turbulent Channel Flows," Journal of Fluid Mechanics, Vol. 475, 2003. doi:10.1017/s002211200200277x. (Also DOI: 10.1063/1.2337733)

Series of excellent LES simulations

Gloerfelt, X. and Berland, G., "Turbulent Boundary-Layer Noise: Direct Radiation at Mach Number 0.5," Journal of Fluid Mechanics, Vol. 723, 2013, pp. 318–351. doi:10.1017/jfm.2013.134.



Mathematical Theory



Modeling Goals and Mathematical Overview

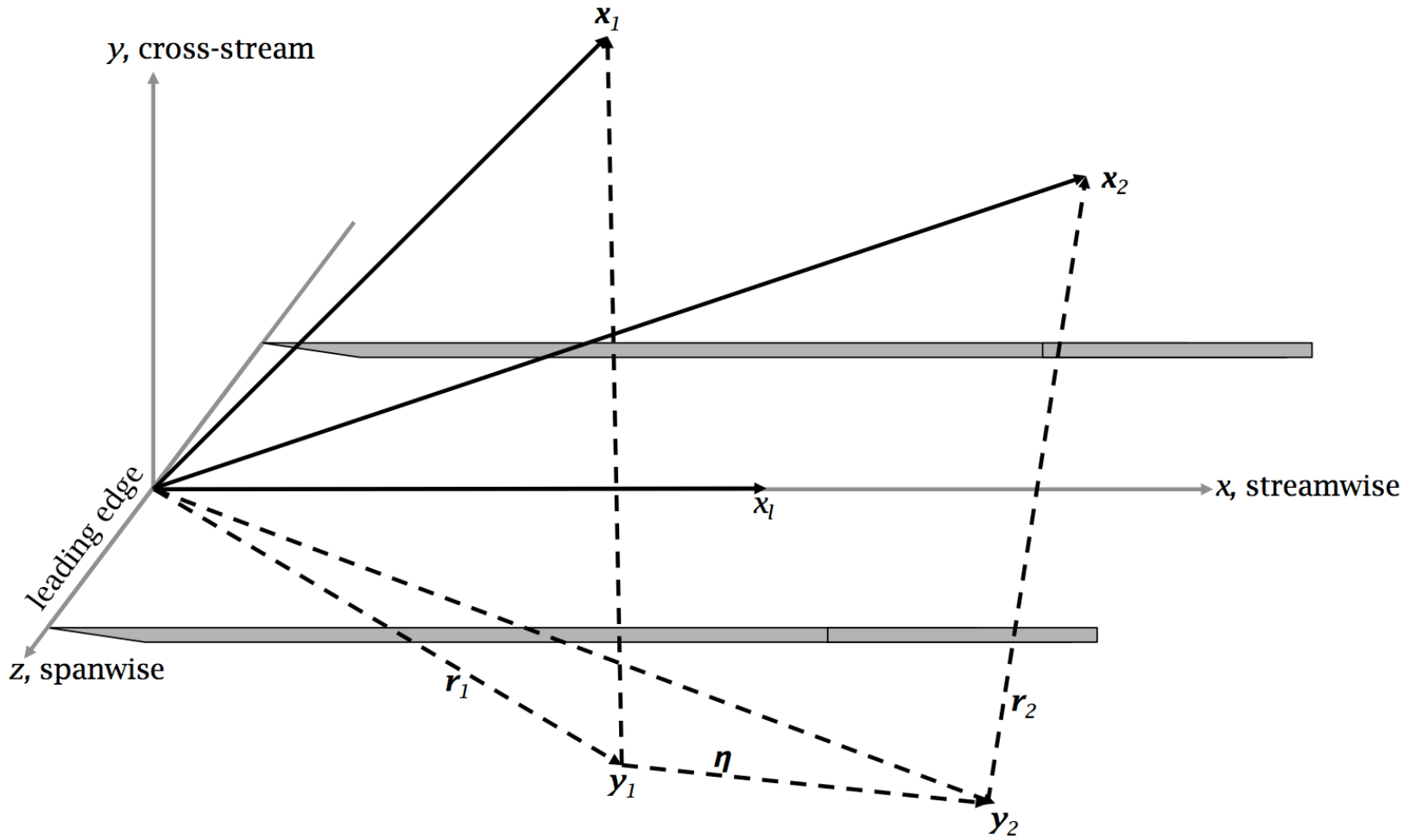
Modeling Goals

- Predict acoustic auto-spectrum and coherence of TBL
- Closed-form mathematical model
- No numerical solution (no CFD, but preserve opportunity)
- Validated for range of subsonic Mach numbers

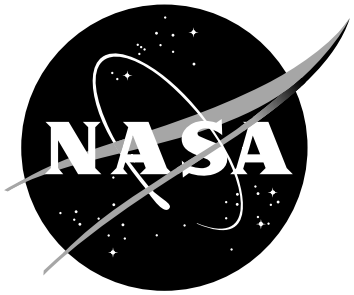
Mathematical Overview

- Start with cross-spectral acoustic analogy
- Evaluation accounting for boundary
- Propose models for arguments
  - Meanflow
  - Turbulent statistics
  - Two-point correlation
- Simplify into integral form and scaling law

Boundary Layer Turbulence Coordinate System



Cross-spectral acoustic analogy uses free-space Green's function
Use concept of mirrored sources to simulate solid boundary



Theoretical Approach

Lighthill's acoustic analogy,

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

One solution with particular assumptions is

$$G\left(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega\right) = \frac{1}{16\pi^2} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \left\{ \overbrace{F_t \overline{\ddot{T}_{ij} \ddot{T}'_{lm}}}^{\text{Far-Field Term}} + \overbrace{M_t \overline{\dot{T}_{ij} \dot{T}'_{lm}}}^{\text{Mid-Field Term}} + \overbrace{N_t \overline{T_{ij} T'_{lm}}}^{\text{Near-Field Term}} \right\} \times \exp\left[-i\omega\left(\tau + \frac{|\boldsymbol{x}_1 - \boldsymbol{y}_1|}{c_\infty} - \frac{|\boldsymbol{x}_2 - \boldsymbol{y}_2|}{c_\infty}\right)\right] d\tau d\boldsymbol{\eta}' d\boldsymbol{\eta}$$

where,

$$r = |\boldsymbol{x} - \boldsymbol{y}| + \boldsymbol{M}_\infty \cdot (\boldsymbol{x} - \boldsymbol{y}) \qquad \text{and} \qquad \boldsymbol{y} = \boldsymbol{\eta} - c_\infty \boldsymbol{M}_\infty t + \boldsymbol{M}_\infty |\boldsymbol{x} - \boldsymbol{y}|$$

Miller, S. A. E., "Prediction of Near-Field Jet Cross Spectra," AIAA Journal, Vol. 53, No. 8, 2015, pp. 2130–2150. doi:10.2514/1.J053614.

Boundary Layer Model Statistics Visualized

Lighthill stress tensor

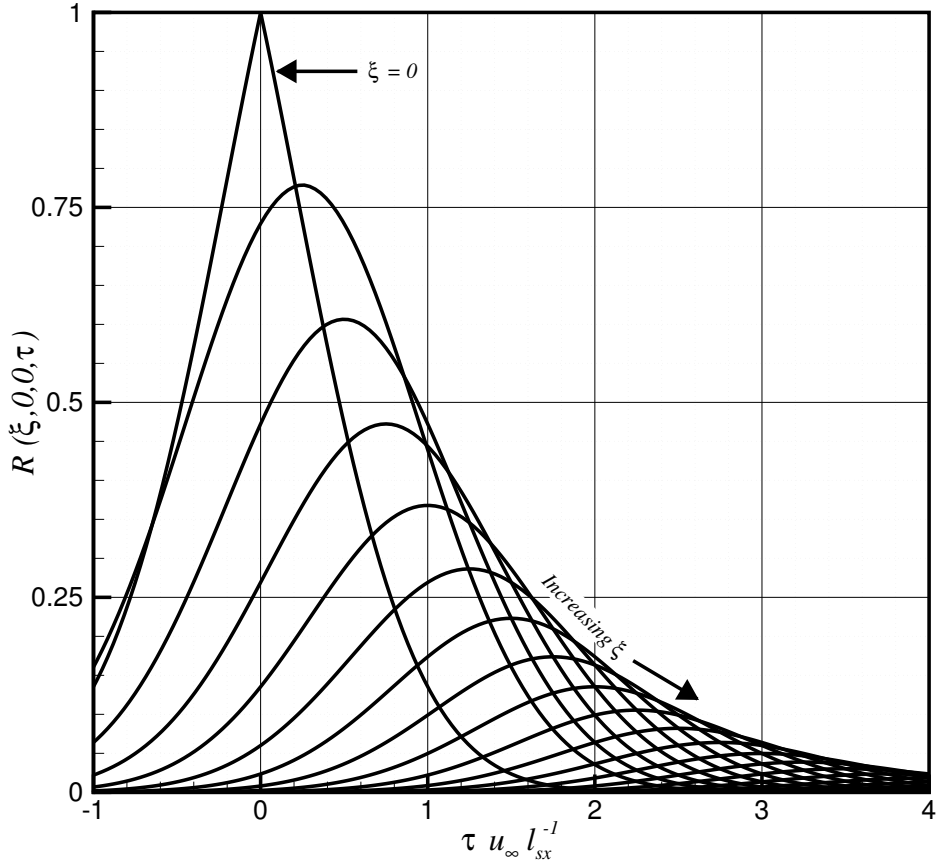
$$\overline{T_{ij}T'_{lm}} \approx \boldsymbol{R}_{ijlm}$$

One potential separable model

$$\boldsymbol{R}_{ijlm} = A_{ijlm}R$$

Coefficient matrix

$$A_{ijlm} \approx \mathcal{P}_f \overline{\rho}\ \overline{\rho}' \left(\overline{u_i u_j}\ \overline{u'_l u'_m}\right)$$



Normalized two-point correlation

$$R = \exp\left[-\frac{(\xi-\overline{u}\tau)^2}{l_{sx}^2}\right]\exp\left[-\frac{(1-\tanh[\alpha|\xi|])|\xi-\overline{u}\tau|}{l_{sx}}\right]\exp\left[-\frac{|\xi|}{l_{sx}}\right]\exp\left[-\frac{|\eta|}{l_{sy}}\right]\exp\left[-\frac{|\zeta|}{l_{sz}}\right]$$

Validation is on-going but captures trends of measurements

See measurements of Naka, Y., Stanislas, M., Foucaut, J. M., Coudert, S., Laval, J. P., and Obi, S., "Space-Time Pressure-Velocity Correlations in a Turbulent Boundary Layer," Journal of Fluid Mechanics, Vol. 771, 2015, pp. 624–675. doi:10.1017/jfm.2015.158.

Velocity Profile

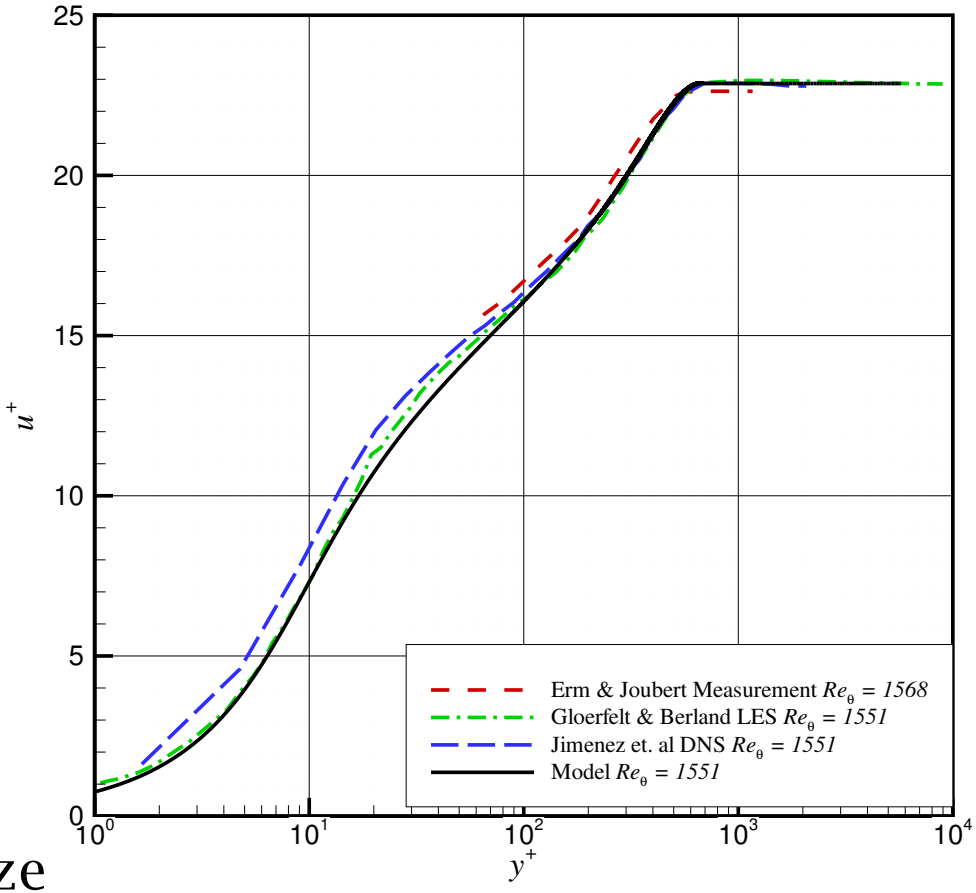
Composite profile of Musker combined with Coles law of the wake

$$u^+_{\text{incompressible}} = \frac{1}{\kappa}\log\left[\frac{y^+ - c_a}{-c_a}\right] + \frac{c_R^2}{(c_a(4c_\alpha - c_a))}$$
$$+\left\{\frac{c_R^2}{(c_a(4c_\alpha - c_a))}\log\left[-\frac{c_a\left((y^+ - c_\alpha)^2 + c_\beta^2\right)^{1/2}}{c_R(y^+ - c_a)}\right]\right.$$
$$\left.+\frac{c_\alpha}{c_\beta}(4c_\alpha + 5c_a)\left(\arctan\left[\frac{(y^+ - c_\alpha)}{c_\beta}\right] + \arctan\left[\frac{c_\alpha}{c_\beta}\right]\right)\right\}$$
$$+\frac{\Pi}{\kappa}\left(1 - \cos\left[\frac{\pi y}{\delta}\right]\right)$$

Huang and Coleman van Driest transform (compressibility)

$$u^+ = \frac{u_\infty}{2u_\tau a^2}\left(Q\sin\left[\frac{au_\tau}{u_\infty}u^+_{\text{incomp.}} - \arcsin\left[bQ^{-1}\right]\right] + b\right)$$

Temperature given by approach of Walze



Gloerfelt, X. and Berland, G., "Turbulent Boundary-Layer Noise: Direct Radiation at Mach Number 0.5," Journal of Fluid Mechanics, Vol. 723, 2013, pp. 318–351. doi:10.1017/jfm.2013.134.

Erm, L. P. and Joubert, P. N., "Low-Reynolds-Number Turbulent Boundary Layers," Journal of Fluid Mechanics, Vol. 230, 1991, pp. 1–44. doi:10.1017/s0022112091000691.

Jimenez, J., Hoyas, S., Simens, M. P., and Mizuno, Y., "Turbulent Boundary Layers and Channels at Moderate Reynolds Numbers," Journal of Fluid Mechanics, Vol. 657, 2010, pp. 335–360. doi:10.1017/S0022112010001370.

Boundary Layer Reynolds Stress Modeling

Streamwise velocity component root mean square

$$\frac{\overline{u'u'}^{1/2}}{u_\tau}\left(0.45\exp\left[-\log[M_\infty+2]+2.1\right]\right)^{-1}$$

$$=\begin{cases} c_4\exp\left[-\frac{(\log[y\delta^{-1}]-\log[c_1])^2}{2c_2^2}\right]+c_8\exp\left[-\frac{(\log[y\delta^{-1}]-\log[c_5])^2}{2c_6^2}\right] & \text{for } \log[y\delta^{-1}]-\log[c_1]\leq 0 \\ c_4\exp\left[-\frac{(\log[y\delta^{-1}]-\log[c_1])^2}{2c_3^2}\right]+c_8\exp\left[-\frac{(\log[y\delta^{-1}]-\log[c_5])^2}{2c_7^2}\right] & \text{for } \log[y\delta^{-1}]-\log[c_1]>0 \end{cases}$$

Cross-stream velocity component root mean square

$$\frac{\overline{v'v'}^{1/2}}{\overline{u'u'}^{1/2}}=1-c_{v3}\exp\left[-\frac{\log[y\delta^{-1}-c_{v1}]^2}{2c_{v2}^2}\right]+\left(1-c_{v6}\exp\left[-\frac{\log[y\delta^{-1}-c_{v4}]^2}{2c_{v5}^2}\right]\right)-1$$

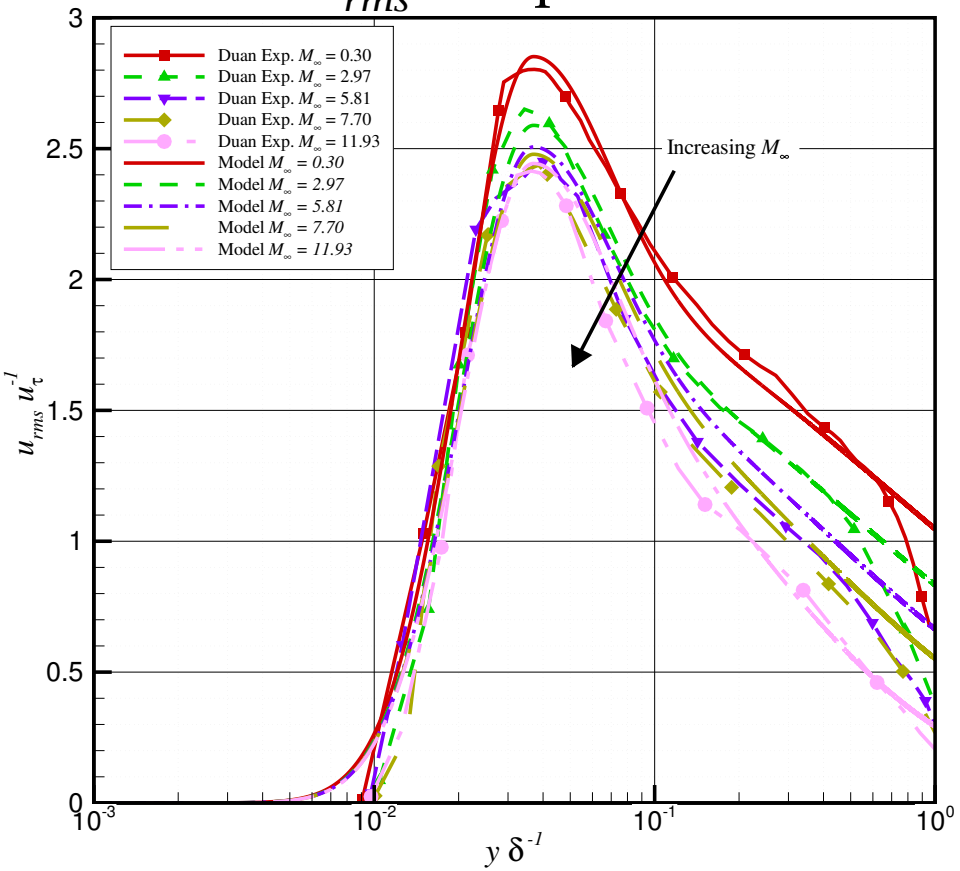
- Similar models for cross-stream component
- Off-diagonal terms approximated with eddy viscosity model
- Length scales adopted from Efimtsov, but in three-dimensions

$$l_s=a_4\delta\left[\left(\frac{a_1 2\pi f}{u_c}\right)^2+\frac{a_2^2}{\left(\frac{2\pi f\delta}{u_\tau}\right)^2+\left(\frac{a_2}{a_3}\right)^2}\right]^{-\frac{1}{2}}$$

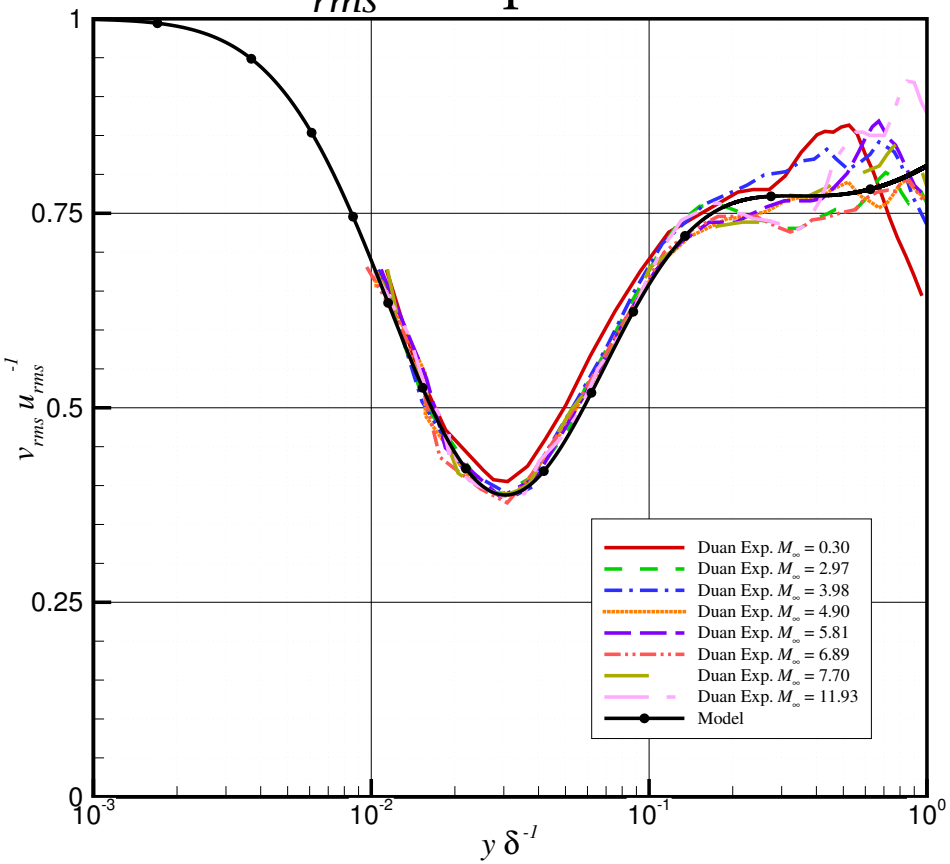
Boundary Layer Reynolds Stress Modeling

Self-similarity (nearly) collapse of the TBL Reynolds stress
Summation of two log-normal distributions

u_{rms} component



v_{rms} component



Duan, L. Beekman, I. and Martin, M. P., "Direct Numerical Simulation of Hypersonic Turbulent Boundary Layers. Part 3. Effect of Mach number," Journal of Fluid Mechanics, Vol. 672, 2011, pp. 245–267. doi:10.1017/s0022112010005902.

Boundary Layer Noise Prediction Models

Cross-spectral model

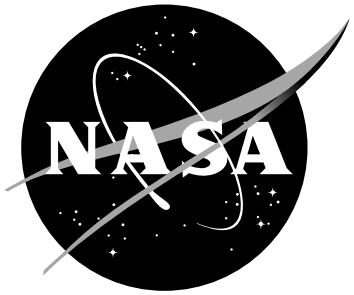
$$G\left(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega\right) = \frac{1}{16\pi^2} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} A_{ijlm} \left\{F_t \mathcal{I}_{\tau 4} + M_t \mathcal{I}_{\tau 2} + N_t \mathcal{I}_{\tau 0}\right\} \times \exp\left[-i\omega\left(\frac{r}{c_\infty} - \frac{r'}{c_\infty}\right)\right] d\boldsymbol{\eta}' d\boldsymbol{\eta}.$$

Scaling of spectral density of acoustic pressure

$$S \propto c_f^2 \overline{\rho}^2 \left(\frac{\rho_\infty}{\rho_w}\right)^2 u_\infty^4 l_{sx} l_{sy} l_{sz} \tau_s \left\{\frac{\overline{u}^4}{c_\infty^4 r^2 l_{sx}^4} + \frac{\overline{u}^2}{c_\infty^2 r^4 l_{sx}^2} + \frac{1}{r^6}\right\} V$$

A simplified form

$$S_{\text{far-field}} \propto \frac{c_f^2}{c_\infty^4 r^2} \frac{\tau_s}{\delta} \left(\frac{\overline{\rho}\rho_\infty}{\rho_w}\right)^2 u_\infty^8 V$$



Results

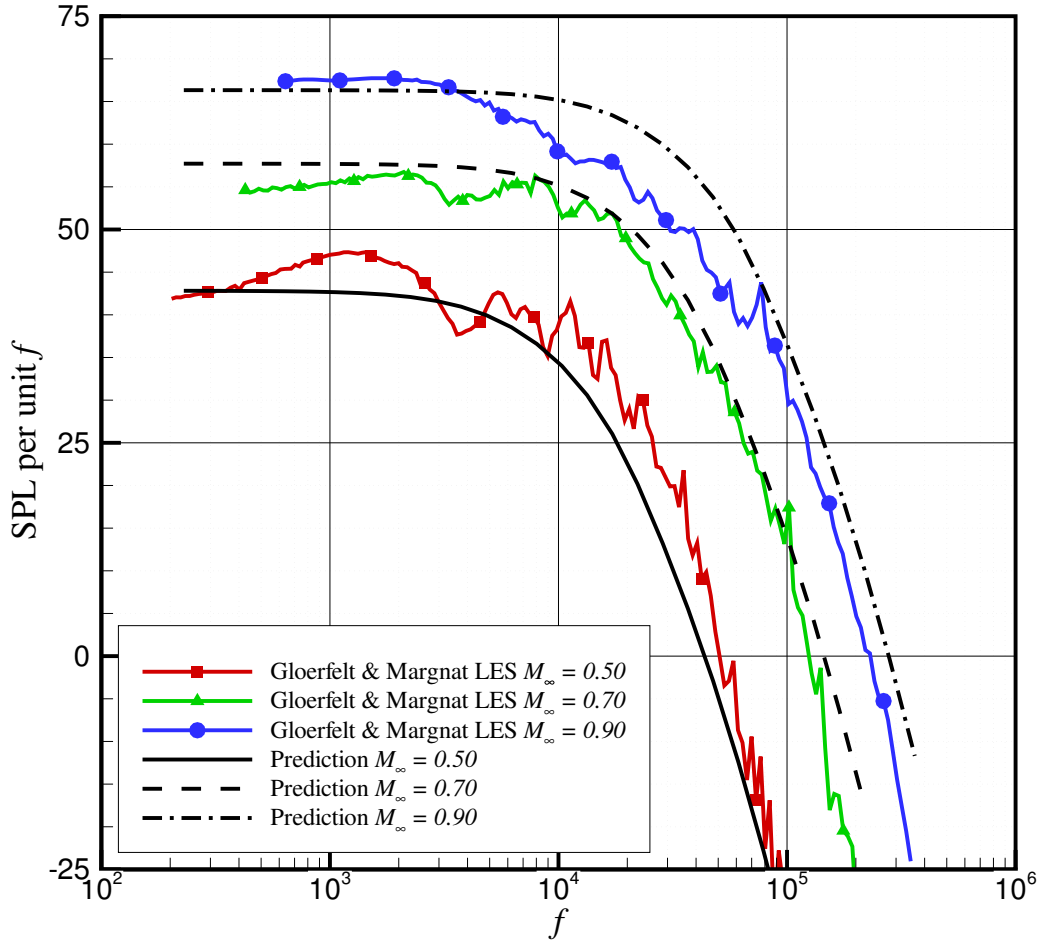
Turbulent Boundary Layer Noise Prediction

Comparisons of predictions with LES

$M_\infty = 0.50, 0.70, 0.90$

$x_2 / \delta \sim 14$
$x_1 / \delta \sim 75$

Generally good agreement between predictions and LES



LES data from Gloerfelt, X. and Margnat, F., "Effect of Mach number on Boundary Layer Noise," AIAA Paper 2014-3291, 2014. doi:10.2514/6.2014-3291.

$\mathcal{M}_\infty$	$\mathcal{R}e_x$	u_∞ ms^{-1}	δ m	c_f	u_τ	x_l m	τ_w	$y+$	$\mathcal{R}e_\theta$	k_{max} m^{2}s^{-2}
0.50	885500	171.6	1.195×10^{-3}	4.034×10^{-3}	7.705	0.0780	071.5	1.960×10^{-6}	1511	252
0.70	773000	240.2	7.526×10^{-4}	4.115×10^{-3}	10.90	0.0486	143.1	1.386×10^{-6}	1322	465
0.90	820000	308.8	6.183×10^{-4}	4.080×10^{-3}	13.95	0.0401	234.4	1.083×10^{-6}	1396	765

Scaling of Turbulent Boundary Layer Noise

Analysis of governing equation shows acoustic energy scales as u^8

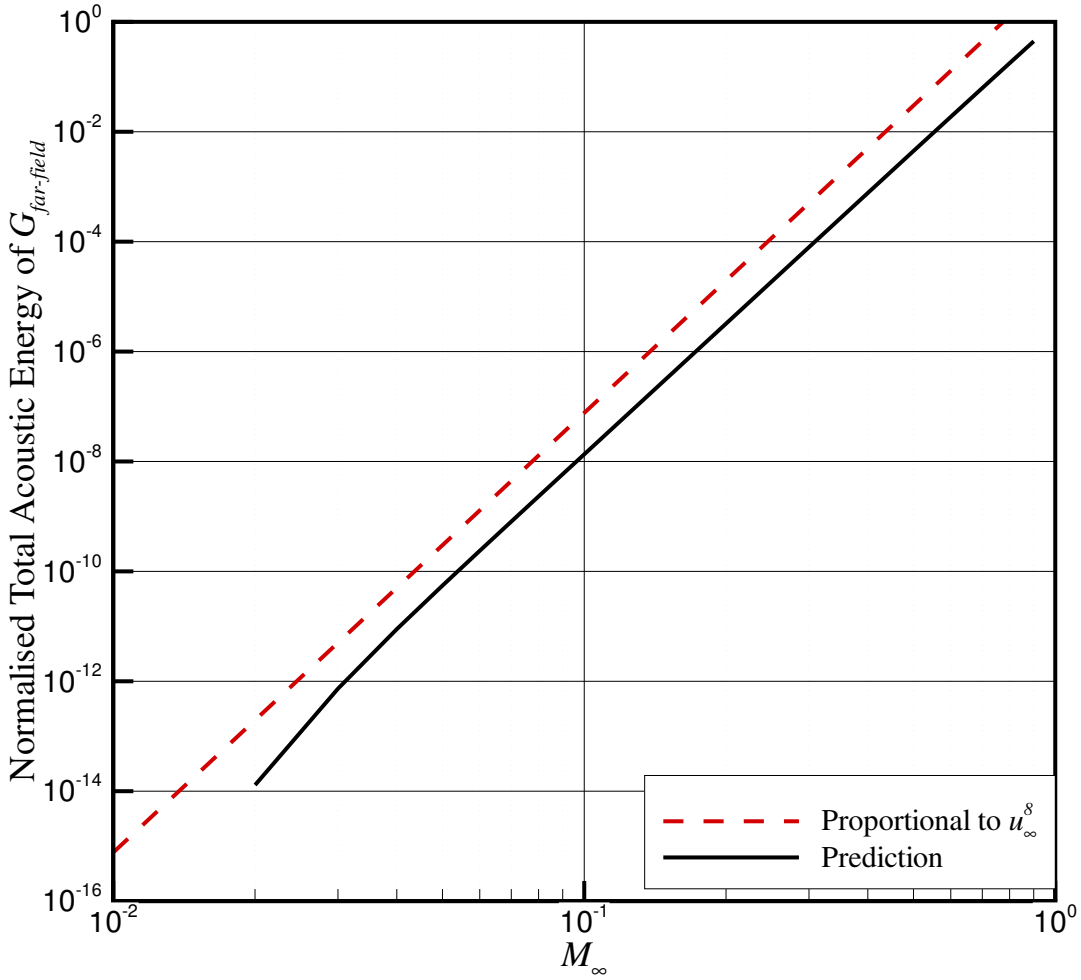
$$S_{\text{far-field}} \propto \frac{c_f^2}{c_\infty^4 r^2} \frac{\tau_s}{\delta} \left(\frac{\overline{\rho}\rho_\infty}{\rho_w} \right)^2 u_\infty^8 V$$

Investigate total acoustic energy with Mach number

$Re_x = 10^6$

Variation of M_∞ from 0 to 1

Clear u^8 trend but arguments can be made for u^7 depending on assumptions

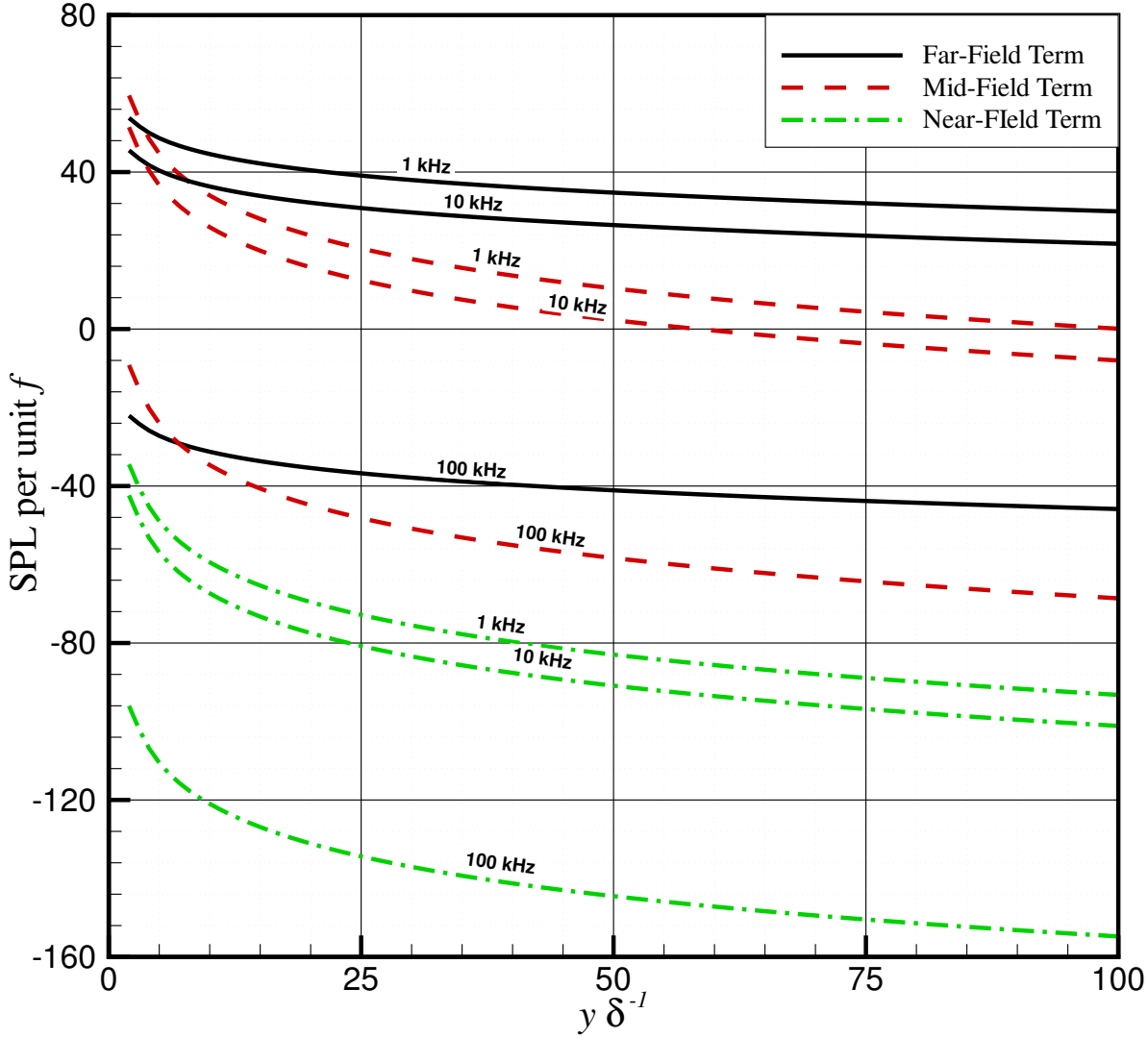


Turbulent Boundary Layer Acoustic Near-Field Predictions

Investigation of near-field, mid-field, and far-field contributions

$M_\infty = 0.50$
$Re_x = 885500$
$u_\tau = 7.70$
$y^+ = 1.96 \times 10^{-6}$ m
$\delta = 1.195 \times 10^{-3}$ m

Observers vary from 1.004δ to 68δ

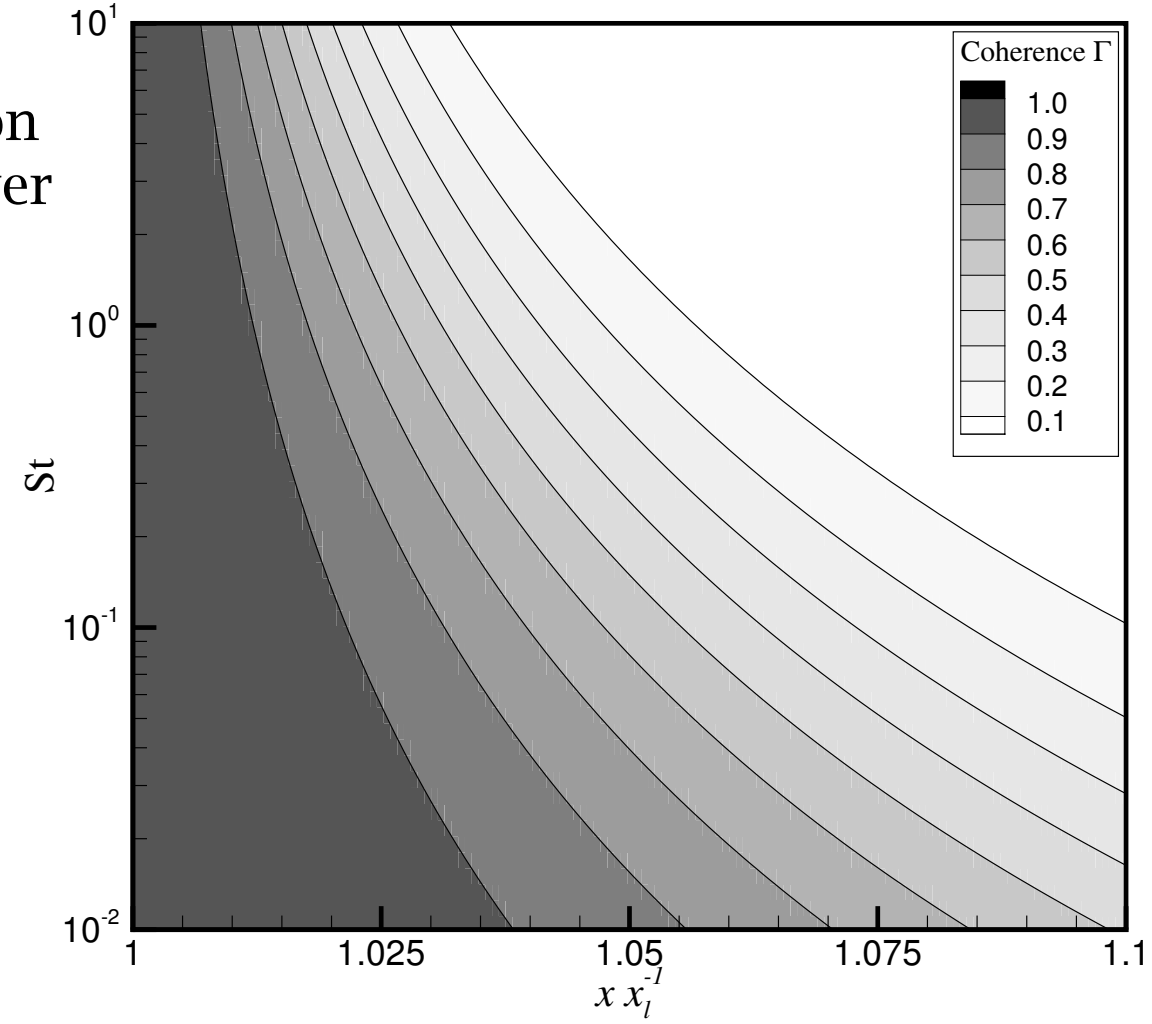


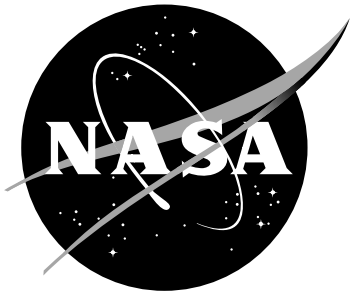
Spatial Coherence

Streamwise probe separation within the boundary layer

$M_\infty = 0.30$
$Re_x = 885500$
$u_\tau = 4.582$
$y^+ = 3.295 \times 10^{-6}$ m
$\delta = 2.2295 \times 10^{-3}$ m
$x_l = 0.14668$ m

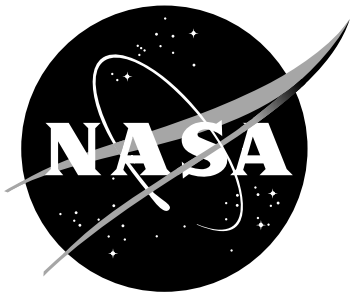
Coherence follows source model





Contemporary Challenges

Accounting for pressure gradient, porosity, and roughness



Pressure Gradient

Industrial flows contain strong pressure gradients within the turbulent boundary layer

Excellent review article and non-dimensional p gradient

Kovasznay, L. S. G., "The Turbulent Boundary Layer," Annual Review of Fluid Mech., Vol. 2, No. 1, 1970, pp. 95–112. doi:10.1146/annurev.fl.02.010170.000523.

$$K = \nu(\rho u_\infty^3)^{-1}\partial p/\partial x$$

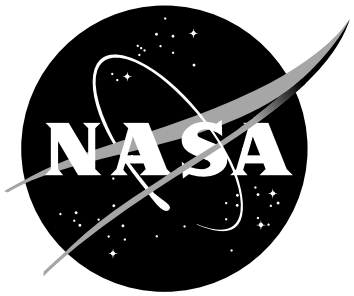
Velocity profiles characterized by K possess partial similarity

Kline, S. J., Reynolds, W. C., Schraub, F. A., and Runstadler, P. W., "The Structure of Turbulent Boundary Layers," Journal of Fluid Mechanics, Vol. 30, No. 4, 1967, pp. 741–773. doi:10.1017/s0022112067001740.

Power law collapses boundary layer in pressure gradient (maybe possible)

Castillo, L., "Similarity Analysis of Turbulent Boundary Layers," State University of New York at Buffalo, Ph.D. Dissertation, 1977.

No acceptable composite profile for turbulent statistics as function of K



Permeability

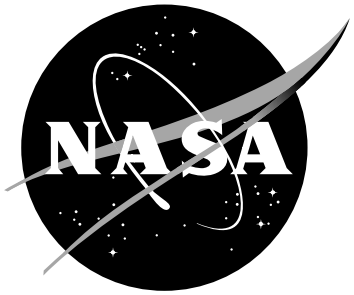
Permeability or porosity of wall has significant effect on the turbulence and subsequent radiated noise

Attempted to alter the coefficients based on measurements in the logarithmic law region

Manes, C., Poggi, D., and Ridolfi, L., "Turbulent Boundary Layers Over Permeable Walls: Scaling and Near-Wall Structure," Journal of Fluid Mechanics, Vol. 687, 2011, pp. 141–170. doi:10.1017/jfm.2011.329

$$u^+ = \kappa^{-1} \log\left[(y+d)(z_0)^{-1}\right] \qquad \mathcal{R}e_{K_p} = K_p^{1/2} u_\tau \nu^{-1}$$

Need a composite profile and statistics that captures Re_{kp} or another approach



Roughness

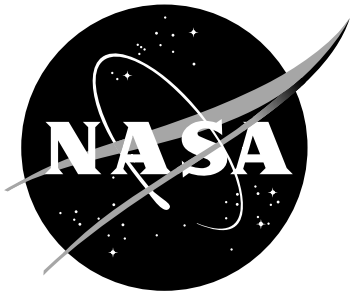
Wall roughness can sometimes impact intensity of radiation

Measurements of noise from pipes with roughness
Hersh, A., "Experimental Investigation of Surface Roughness Generated Flow Noise," 8th AIAA Aeroacoustics Conference, AIAA Paper 1983-768, 1983. doi:10.2514/6.1983-786.

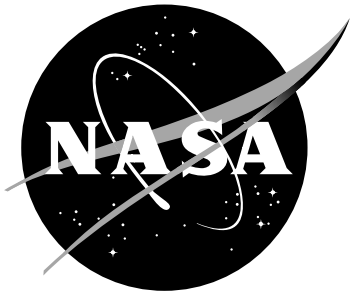
$$\omega r_\eta u_\tau^{-1} < 5$$

Developed approach to account for roughness where turbulent statistics are not significantly altered
Howe, M. S., "Surface Pressures and Sound Produced by Turbulent Flow Over Smooth and Rough Walls," Journal of the Acoustical Society of America, Vol. 90, No. 2, 1991, pp. 1041–1047. doi: 10.1121/1.402292.

Turbulent boundary layer statistics must depend on r_η or another roughness parameter



Summary and Conclusion



Summary and Conclusion

Boundary Layer Turbulence

- One canonical fluid flow
- Noise contribution in wind tunnels and airframe noise

Mathematical Theory

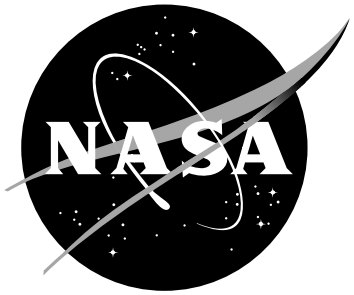
- Cross-spectral acoustic analogy
- Related sources to boundary layer statistics

Predictions

- Source statistics and acoustic predictions agree with measurement and numerical simulation

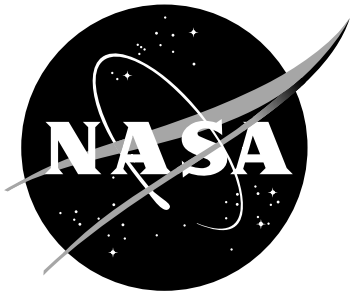
Research Ongoing

- Modeling source statistics that are dependent on pressure gradient, porosity, and roughness



Questions

Thank You



Extra Slides