Direct numerical simulation and parametric study of the noise generated from particle dispersion in decaying homogeneous isotropic turbulent flow

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Acknowledgement

This research is supported by the Florida Space Research Initiative (SRI) - OR-DRPD-SRI2019: Prediction and Reduction of Noise from Rockets to Eliminate Failure and Fatigue.
Outline

- Introduction
- Numerical Methods
  - Governing Equations
  - Acoustic Analogy
- Result and Discussion
  - Problem Definition
- Root-mean-square Acoustic Pressure
- Acoustic Source Terms
- Summary and Future Work
Introduction

- Decaying isotropic turbulence laden with solid particles
- Two-way coupling of gas and particles modifies turbulence
- Effect of physical parameters of particles on noise generation is not clear

Workflow of Hybrid DNS/CAA

Governing Equations

- Navier-Stokes equations

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{W} d\Omega + \oint_{\partial \Omega} \mathbf{F}_c dS = \oint_{\partial \Omega} \mathbf{F}_p dS + \int_{\Omega} \mathbf{Q} d\Omega$$

where source terms are

$$\mathbf{Q} = \begin{bmatrix} 0, f_{p,x}, f_{p,y}, f_{p,z}, E_p \end{bmatrix}^T$$

$$\mathbf{f}_p = - \sum m_p (\mathbf{V} - \mathbf{V}_p)$$

$$E_p = \sum \left[ \mathbf{f}_p \cdot (\mathbf{V}_p - \mathbf{V}) - \frac{m_p C_{p,p} (T - T_p)}{\tau_\theta} \right]$$

- Point-Particle equations

- Evolution Equations

$$\frac{d}{dt} \mathbf{x}_p = \mathbf{V}_p , \quad \frac{d}{dt} \mathbf{V}_p = \frac{\mathbf{V}_p - \mathbf{V}}{\tau_u} , \quad \frac{d}{dt} T_p = \frac{T - T_p}{\tau_\theta}$$

- Time Scales

$$\tau_u = \frac{\rho_p d_p^2}{18 \mu f_u(Re)} , \quad \tau_\theta = \frac{C_{p,p} \rho_p d_p^2}{12 k f_\theta(Re)}$$

- Numerical solver – RocfluidMP
Two-Phase Acoustic Analogy

- Crighton and Ffowcs Williams\[^1\] Acoustic Analogy

\[
\left( \frac{\partial^2}{\partial t^2} - c^2_\infty \nabla^2 \right) (\rho - \rho_\infty) = \frac{\partial Q}{\partial t} - \frac{\partial G_i}{\partial x_i} + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

- Monopole: \( Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha) \)  
  **Effect of volume fraction**

- Dipole: \( G_i = F_i + \frac{\partial}{\partial t} \alpha \rho v_i \)  
  **Effect of drag force**

- Quadrupole: \( T_{ij} = (1 - \alpha) \rho v_i v_j + (p_{ij} - c^2_\infty \rho \delta_{ij}) \)

- Solution using Free-Space Green’s Function

\[
4\pi P'(x, t) = \frac{1}{x} \int \frac{\partial}{\partial t} Q \left( y, t - \frac{|x - y|}{c_\infty} \right) dy + \frac{x_i}{x^2 c_\infty} \int \frac{\partial}{\partial t} G_i \left( y, t - \frac{|x - y|}{c_\infty} \right) dy
\]
\[
+ \frac{x_i x_j}{x^2 c^2_\infty} \int \frac{\partial^2}{\partial t^2} T_{ij} \left( y, t - \frac{|x - y|}{c_\infty} \right) dy
\]

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DNS Simulations

- Initial condition is prescribed using Kraichnan’s method [1].

- Schumann and Patterson [2] TKE spectrum
  \[ E(k, 0) = \left( \frac{3}{2} u_0^2 \right) \left( \frac{1}{2\pi} \right) \left( \frac{k}{k_p} \right) \exp \left( - \frac{k}{k_p} \right) \]

- Study the effects of total number and diameter of particle on the generated noise

- DNS of total of 25 cases are performed

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Particles (thousands)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Particle Diameter ((10^{-6} \text{m}))</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
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Acoustic Pressure

- Acoustic pressure is close to the gas case when particle number are low
- Acoustic pressure increase with increasing number and diameter of particles
- Effects of number and diameter of particles increase with more particles
- Effect of Initial condition is only significant at low particle number
Trendlines

- $p_{rms} \sim \alpha^{0.15}$, for $u_0^* = 0.05$ \( \alpha < 10^{-6} \)
- $p_{rms} \sim \alpha^{0.01}$, for $u_0^* = 0.20$
- $p_{rms} \sim \alpha^{2.42}$, for $\alpha > 10^{-6}$

Two different noise generation mechanisms

- Turbulence dominates for $\alpha < 10^{-7}$
- Particle dynamics dominates for $\alpha > 10^{-6}$
Acoustic Pressure

Acoustic pressure of quadrupole sources

\[ 4\pi p_{quad}'(x, t) = \frac{x_i x_j}{x^2} \frac{1}{c_\infty^2} \int \frac{\partial^2}{\partial t^2} T_{ij} \left( y, t - \frac{|x - y|}{c_\infty} \right) dy \]

Effects of number and diameter of particles are small

Effect of \( u_0^* \) is significant
Acoustic Pressure

- Quadrupole source is largely not dependent on the increasing volume fraction
- There is slight decrease at volume fraction around $10^{-5}$
- Acoustic pressure increases at high volume fraction
- The effect of initial velocity is significant
Acoustic Pressure

- Acoustic pressure of monopole sources

\[ 4\pi p_{\text{mono}}' (x, t) = \frac{1}{x} \int \frac{\partial}{\partial t} Q \left( y, t - \frac{|x - y|}{c_\infty} \right) dy \]

where \( Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha) \)

- Acoustic pressure of monopole and dipole source are similar in trends

- Effects of number and diameter of particles are independent on each other
Acoustic Pressure

- Trendlines
  - Monopole $p_{rms} \sim \alpha^{2.49}$
  - Dipole $p_{rms} \sim \alpha^{1.74}$

- Monopole source is comparable with dipole source.

- Dusty gas assumption is not valid.

- Quadrupole source dominate for $\alpha < 10^{-6}$
We perform DNS isotropic turbulence laden with particle in 25 cases with varying total number and diameter of particles.

We apply Crighton and Ffowcs Williams acoustic analogy for two-phase flow.

We compare the change of root-mean-square acoustic pressure w.r.t. total number, diameter, and volume fraction of particles.

Quadrupole source dominates for low volume fraction.

Monopole and dipole sources are comparable in magnitudes and dominate with increasing volume fraction.

Acoustic pressure scales as $2.5^{\text{th}}$ power of the initial volume fraction of particles.

Future work – we plan to utilize linear forcing and study stationary isotropic turbulence laden with particles.
Thank you!