

# Direct numerical simulation and parametric study of the noise generated from particle dispersion in decaying homogeneous isotropic turbulent flow

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# Acknowledgement

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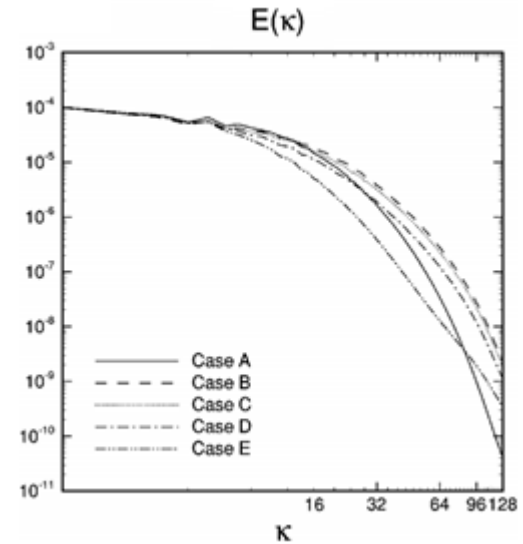
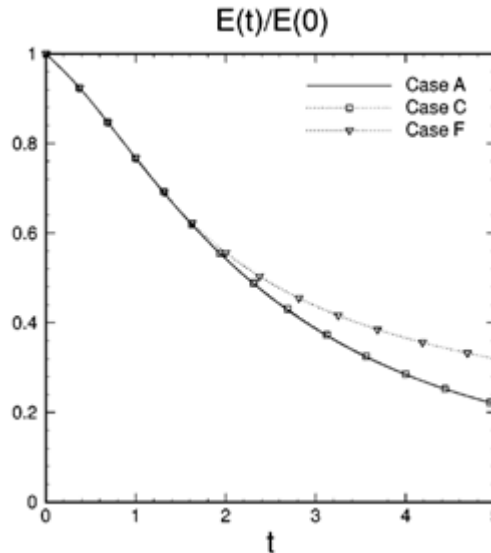


# Outline

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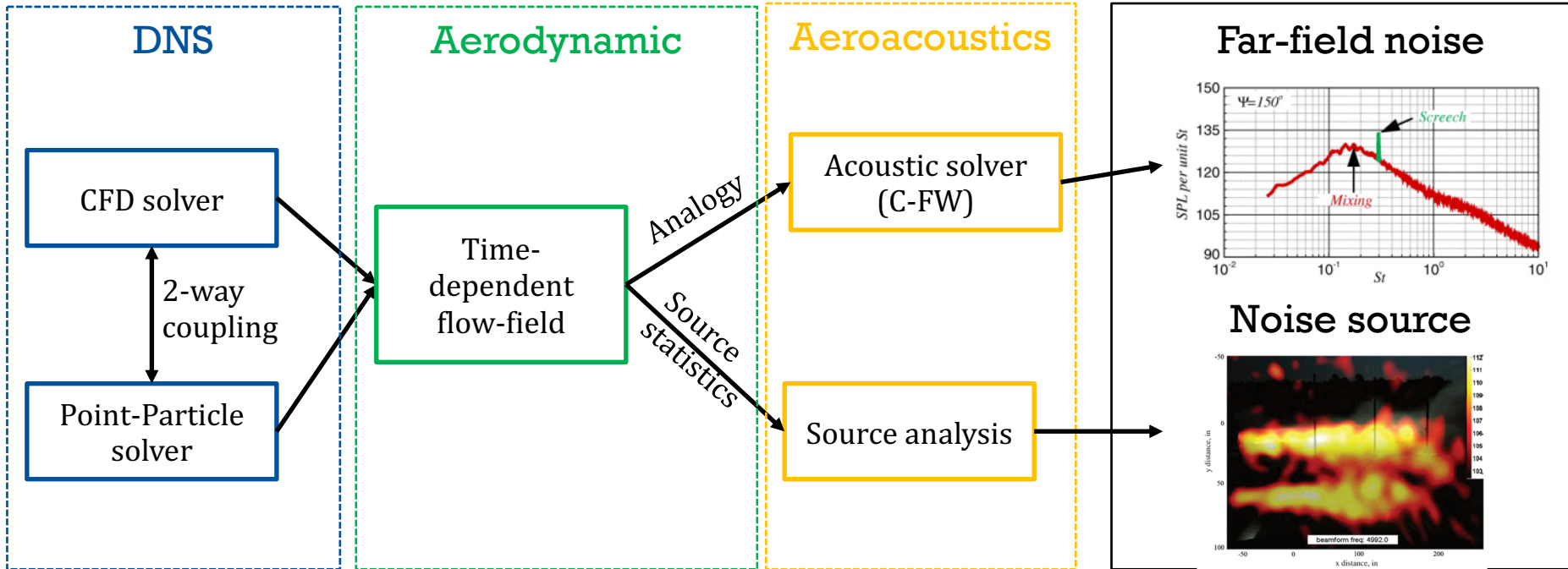
# Introduction

- Decaying isotropic turbulence laden with solid particles
- Two-way coupling of gas and particles modifies turbulence
- Effect of physical parameters of particles on noise generation is not clear



Ferrante, A. and Elghobashi, S., "On the physical mechanisms of two-way coupling in particle-laden isotropic turbulence," *Physics of fluids*, Vol. 15, No. 2, 2003, pp. 315-329.

# Workflow of Hybrid DNS/CAA



[1] Miller, S. A. E., "Toward a Comprehensive Model of Jet Noise Using an Acoustic Analogy," *AIAA Journal*, Vol. 52, No. 10, 2014, pp. 2143–2164. doi:10.2514/1.j052809.

[2] Panda, J., and Mosher, R., "Microphone Phased Array to Identify Liftoff Noise Sources in Model-Scale Tests," *Journal of Spacecraft and Rockets*, Vol. 50, No. 5, 2013, pp. 1002–1012. doi:10.2514/1.A32433.

# Governing Equations

## ■ Navier-Stokes equations

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{W} d\Omega + \oint_{\partial\Omega} \mathbf{F}_c dS = \oint_{\partial\Omega} \mathbf{F}_v dS + \int_{\Omega} \mathbf{Q} d\Omega$$

where source terms are

$$\mathbf{Q} = [0, f_{p,x}, f_{p,y}, f_{p,z}, E_p]^T$$

$$\mathbf{f}_p = - \sum \frac{m_p (\mathbf{V} - \mathbf{V}_p)}{\tau^u}$$

$$E_p = \sum \left[ \mathbf{f}_p \cdot (\mathbf{V}_p - \mathbf{V}) - \frac{m_p C_{p,p} (T - T_p)}{\tau^\theta} \right]$$

## ■ Point-Particle equations

### ■ Evolution Equations

$$\frac{d}{dt} \mathbf{x}_p = \mathbf{V}_p, \quad \frac{d}{dt} \mathbf{V}_p = \frac{\mathbf{V} - \mathbf{V}_p}{\tau^u}, \quad \frac{d}{dt} T_p = \frac{T - T_p}{\tau^\theta}$$

### ■ Time Scales

$$\tau^u = \frac{\rho_p d_p^2}{18\mu f_u(Re)}, \quad \tau^\theta = \frac{C_{p,p} \rho_p d_p^2}{12k f_\theta(Re)}$$

## ■ Numerical solver – RocfluidMP

# Two-Phase Acoustic Analogy

- Crighton and Ffowcs Williams<sup>[1]</sup> Acoustic Analogy

$$\left( \frac{\partial^2}{\partial t^2} - c_\infty^2 \nabla^2 \right) (\rho - \rho_\infty) = \frac{\partial Q}{\partial t} - \frac{\partial G_i}{\partial x_i} + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

- Monopole:  $Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha)$  ← Effect of volume fraction

- Dipole:  $G_i = F_i + \frac{\partial}{\partial t} \alpha \rho v_i$  ← Effect of drag force

- Quadrupole:  $T_{ij} = (1 - \alpha) \rho v_i v_j + (p_{ij} - c_\infty \rho \delta_{ij})$

- Solution using Free-Space Green's Function

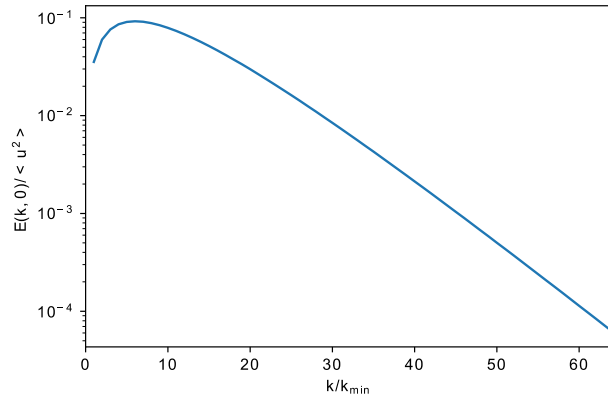
$$4\pi p'(\mathbf{x}, t) = \frac{1}{x} \int \frac{\partial}{\partial t} Q \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{c_\infty} \right) dy + \frac{x_i}{x^2} \frac{1}{c_\infty} \int \frac{\partial}{\partial t} G_i \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{c_\infty} \right) dy + \frac{x_i x_j}{x^2} \frac{1}{c_\infty^2} \int \frac{\partial^2}{\partial t^2} T_{ij} \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{c_\infty} \right) dy$$

[1] Crighton, D. G., "The excess noise field of subsonic jets," Journal of Fluid Mechanics, Vol. 56, No. 4, 1972, pp. 683-694. doi:10.1017/s0022112072002605.

# DNS Simulations

- Initial condition is prescribed using Kraichnan’s method [1].
- Schumann and Patterson [2] TKE spectrum
- Study the effects of total number and diameter of particle on the generated noise
- DNS of total of 25 cases are performed

$$E(k, 0) = \left(\frac{3}{2}u_0^{*2}\right) \left(\frac{1}{2\pi}\right) \left(\frac{k}{k_p^2}\right) \exp\left(-\frac{k}{k_p}\right)$$

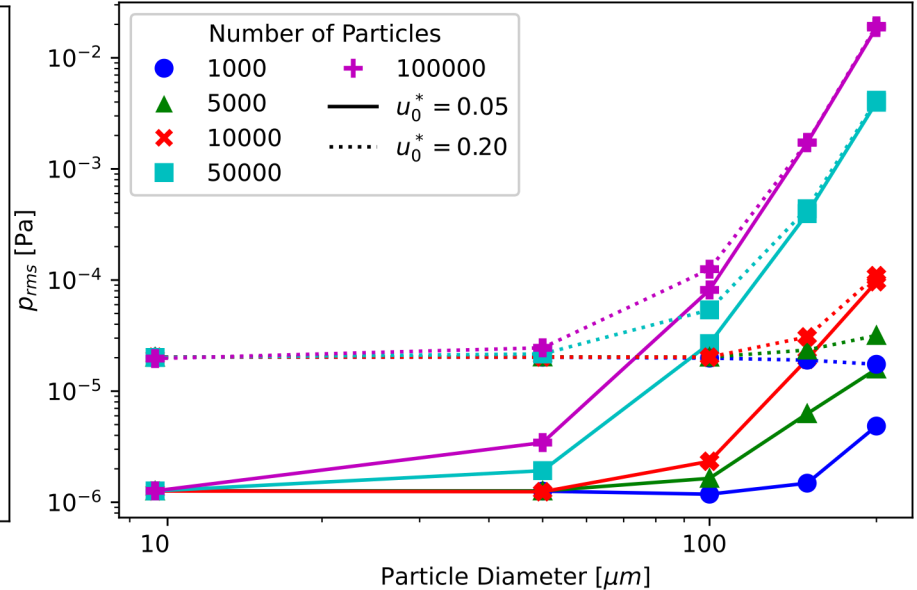
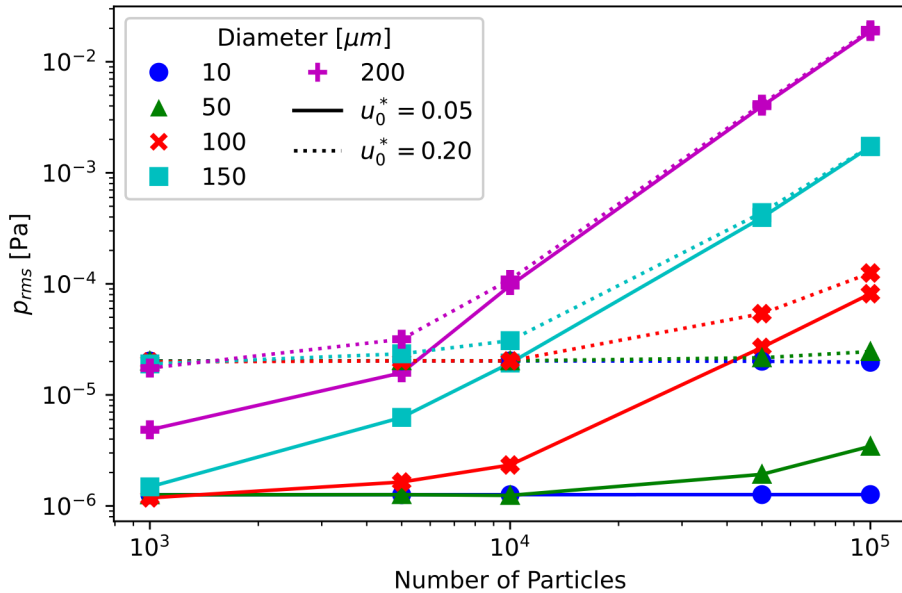


Parameters					
Total Number of Particles (thousands)	1	5	10	50	100
Particle Diameter (10 <sup>-6</sup> m)	10	50	100	150	200

[1] Kraichnan, R. H., “The structure of isotropic turbulence at very high Reynolds numbers,” *Journal of Fluid Mechanics*, Vol. 5, No. 4, 1959, pp. 497–543.  
 [2] Schumann, U., and Patterson, G. S., “Numerical study of pressure and velocity fluctuations in nearly isotropic turbulence,” *Journal of Fluid Mechanics*, Vol. 88, No. 4, 1978, pp. 685–709



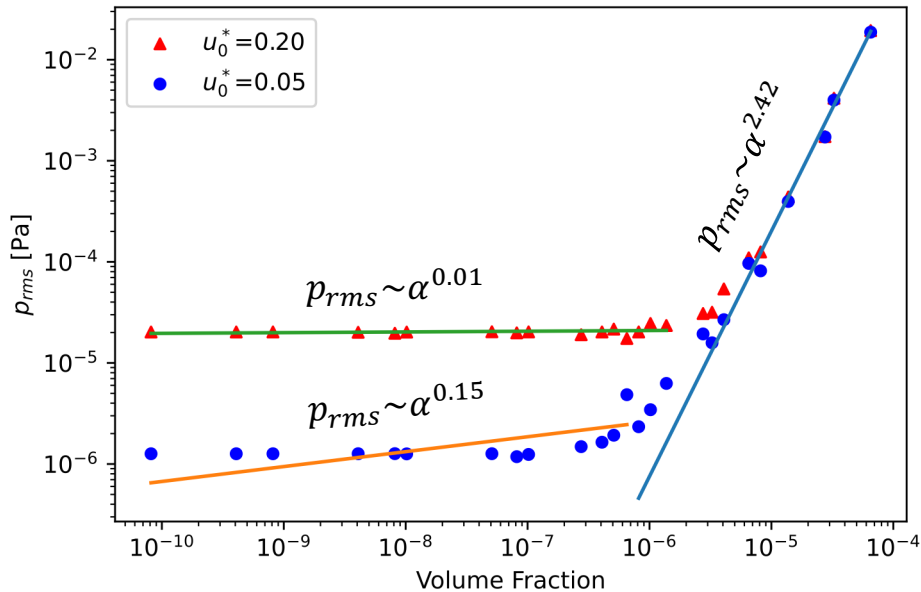
# Acoustic Pressure



- Acoustic pressure is close to the gas case when particle number are low
- Acoustic pressure increase with increasing number and diameter of particles

- Effects of number and diameter of particles increase with more particles
- Effect of Initial condition is only significant at low particle number

# Acoustic Pressure



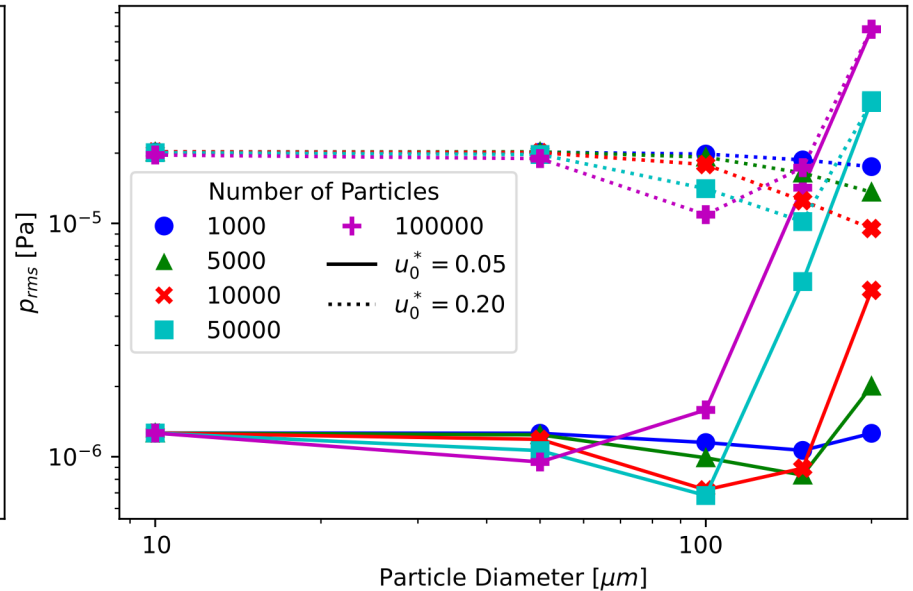
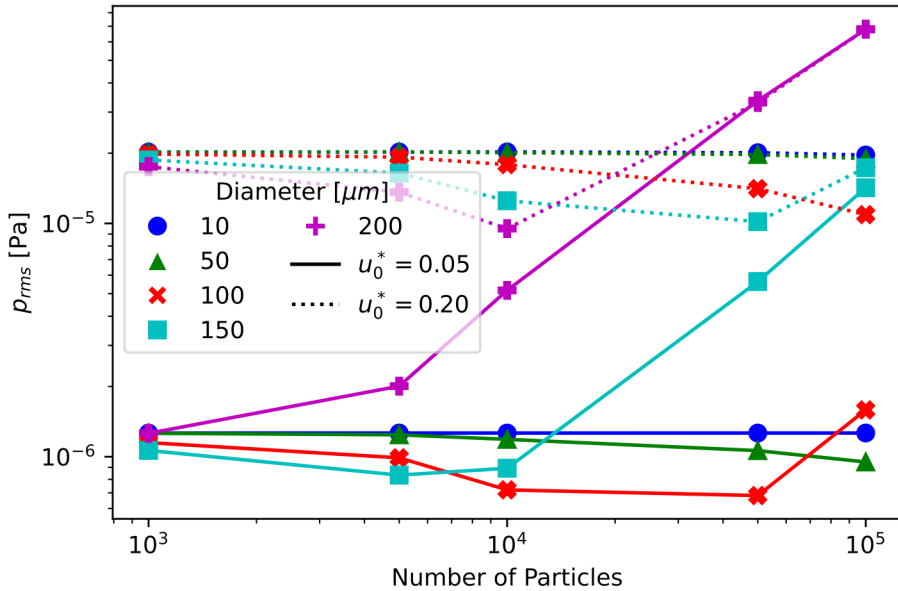
## Trendlines

- $p_{rms} \sim \alpha^{0.15}$ , for  $u_0^* = 0.05$
  - $p_{rms} \sim \alpha^{0.01}$ , for  $u_0^* = 0.20$
  - $p_{rms} \sim \alpha^{2.42}$ , for  $\alpha > 10^{-6}$
- }  $\alpha < 10^{-6}$

## Two different noise generation mechanisms

- Turbulence dominates for  $\alpha < 10^{-7}$
- Particle dynamics dominates for  $\alpha > 10^{-6}$

# Acoustic Pressure

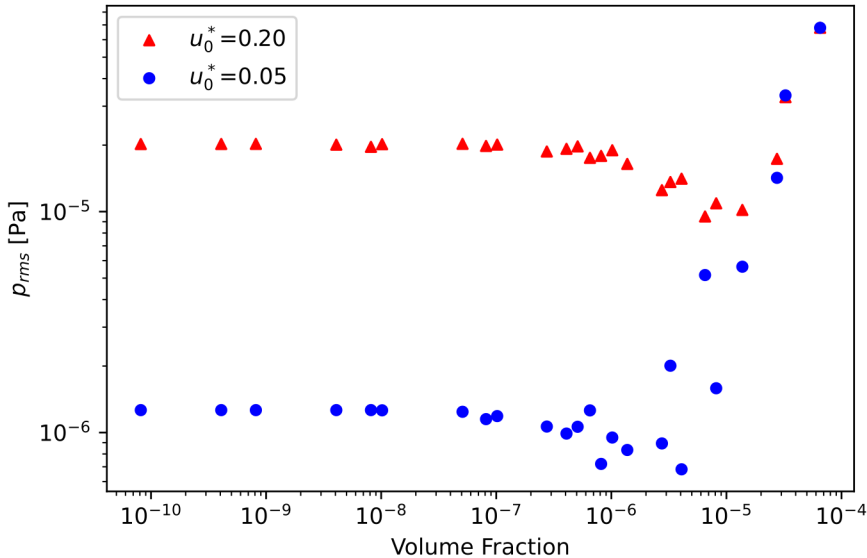


- Acoustic pressure of quadrupole sources

$$4\pi p'_{quad}(\mathbf{x}, t) = \frac{x_i x_j}{x^2} \frac{1}{c_\infty^2} \int \frac{\partial^2}{\partial t^2} T_{ij} \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{c_\infty} \right) dy$$

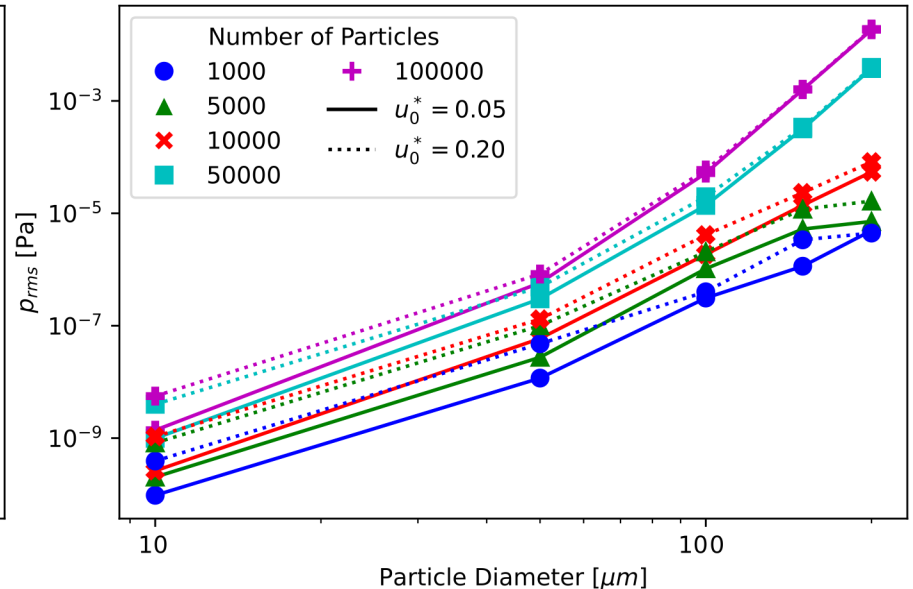
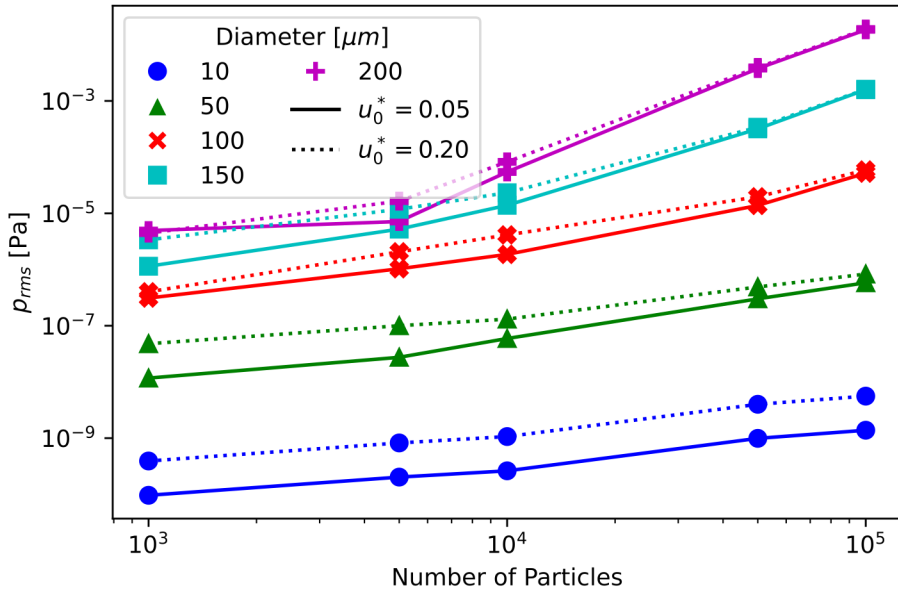
- Effects of number and diameter of particles are small
- Effect of  $u_0^*$  is significant

# Acoustic Pressure



- Quadrupole source is largely not dependent on the increasing volume fraction
- There is slight decrease at volume fraction around  $10^{-5}$
- Acoustic pressure increases at high volume fraction
- The effect of initial velocity is significant

# Acoustic Pressure



## Acoustic pressure of monopole sources

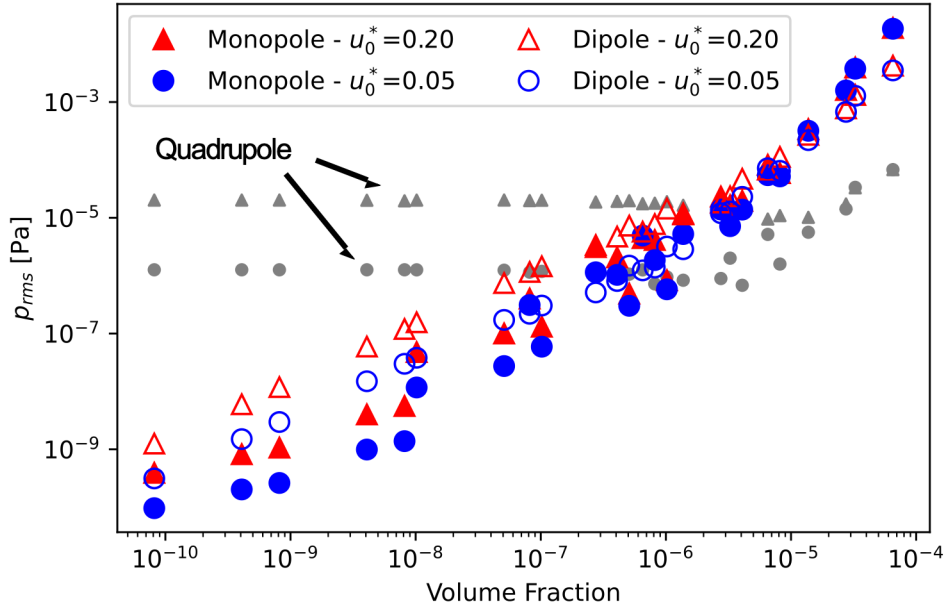
$$4\pi p'_{mono}(x, t) = \frac{1}{x} \int \frac{\partial}{\partial t} Q \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{c_\infty} \right) d\mathbf{y}$$

$$\text{where } Q = -\rho \left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \right) \ln(1 - \alpha)$$

## Acoustic pressure of monopole and dipole source are similar in trends

- Effects of number and diameter of particles are independent on each other

# Acoustic Pressure



- Trendlines
  - Monopole  $p_{rms} \sim \alpha^{2.49}$
  - Dipole  $p_{rms} \sim \alpha^{1.74}$
- Monopole source is comparable with dipole source.
- Dusty gas assumption is not valid.
- Quadrupole source dominate for  $\alpha < 10^{-6}$

# Summary and Future Work

- We perform DNS isotropic turbulence laden with particle in 25 cases with varying total number and diameter of particles.
- We apply Crighton and Ffowcs Williams acoustic analogy for two-phase flow.
- We compare the change of root-mean-square acoustic pressure w.r.t. total number, diameter, and volume fraction of particles.
- Quadrupole source dominates for low volume fraction.
- Monopole and dipole sources are comparable in magnitudes and dominate with increasing volume fraction.
- Acoustic pressure scales as  $2.5^{\text{th}}$  power of the initial volume fraction of particles.
- Future work – we plan to utilize linear forcing and study stationary isotropic turbulence laden with particles.

Thank you!