A Modified Acoustic-Turbulent Scattering Model for Long-Range Propagation with Wind Tunnel Validation

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ASA 2021
December

Acknowledgements

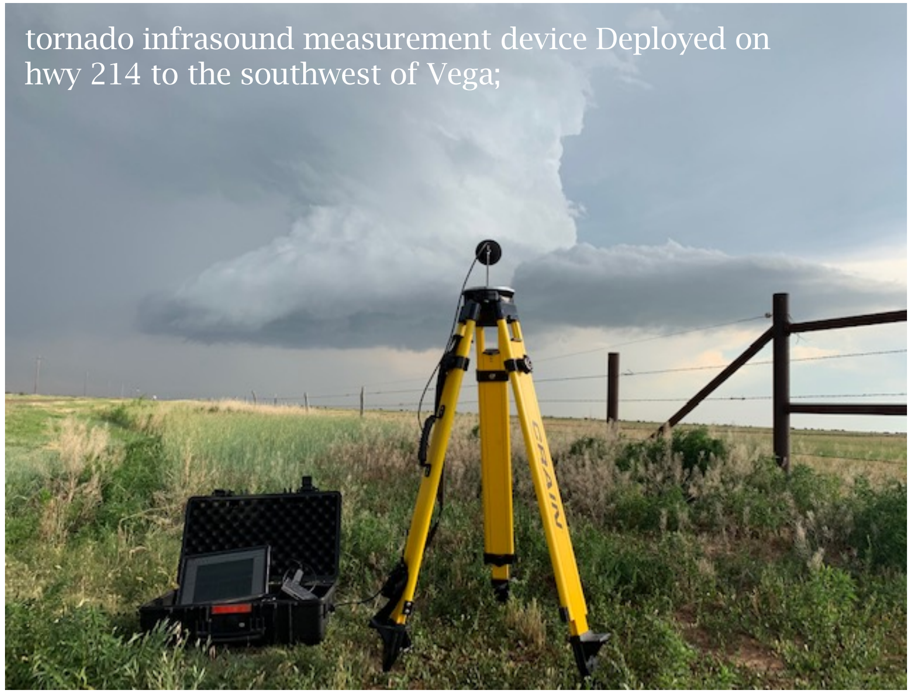
The authors are grateful for continuous funding from NOAA. This program is supported by the NOAA Weather and Air Quality Research program award number NA18OAR4590306.

Outline

- Background
  - Atmospheric Infrasound Propagation
  - Acoustic-Turbulent Interaction Models
- Methodology
  - Turbulent Scattering Model
  - Modification of the Bridging Model
- Wind Tunnel Experiment
  - Experiment Setup
  - Experimental Validation
- Model Behaviors
  - Sensitivity
  - Behavior in Atmospheric Turbulence
- Summary, Conclusions, and Future Work

Introduction

- Infrasound long-range propagation in atmosphere has been studied for the objects like tornado and explosion
- The acoustic-turbulent interaction has been studied by many works such as Lighthill[1], Tartaski[2], and Ostashev and Wilson[3]
- A combination of acoustic ray tracing, generalized Burgers' equation, and turbulent scattering model
- A series of wind tunnel tests are conducted to validate the newly-developed model



Field tornado recording team from our TTU partner.

[1] Lighthill, M. J., "On the Energy Scattered from the Interaction of Turbulence with Sound or Shock Waves," *Mathematical Proceedings of the Cambridge Philosophical Society*,Vol. 49, No. 3, 1953, pp. 531–551

[2] Tatarski, V. I.,*Wave Propagation in a Turbulent Medium*, Dover Publication, 1967

[3] Ostashev, V. E., and Wilson, D. K. (2015). Acoustics in moving inhomogeneous media (CRC Press).

Ostashev and Wilson Scattering Model[1]

Ostashev and Wilson's model is derived by using the Helmholtz-type equation

$$\left[\nabla^2 + \mathrm{k}^2(1+\epsilon) - \left(\nabla \ln\left(\frac{\rho}{\rho_0}\right)\right)\cdot\nabla - \frac{2i}{\omega}\frac{\partial \tilde{v}_i}{\partial x_j}\frac{\partial^2}{\partial x_j \partial x_i} + \frac{2ik}{c_0}\tilde{\boldsymbol{v}}\cdot\nabla\right]\hat{p}\,(\boldsymbol{R}) = \rho(i\omega - \tilde{\boldsymbol{v}})\hat{Q}(\boldsymbol{R})$$

The model can capture the effects of humidity and temperature fluctuation

Obtaining scattering intensity is

$$\langle I_s \rangle = \frac{2\pi k^4 V I_0 n}{R^2}\left[\frac{\beta^2(\theta)\Phi_{\mathrm{T}}(\mathrm{q})}{4T_0^2} + \frac{\beta(\theta)\eta(\theta)\Phi_{\mathrm{CT}}(\mathrm{q})}{2T_0} + \frac{\eta^2(\theta)\Phi_{\mathrm{C}}(\mathrm{q})}{4} + \frac{\cos^2\theta\, n_{0,i} n_{0,j}\Phi_{ij}(q)}{c_0^2}\right]$$

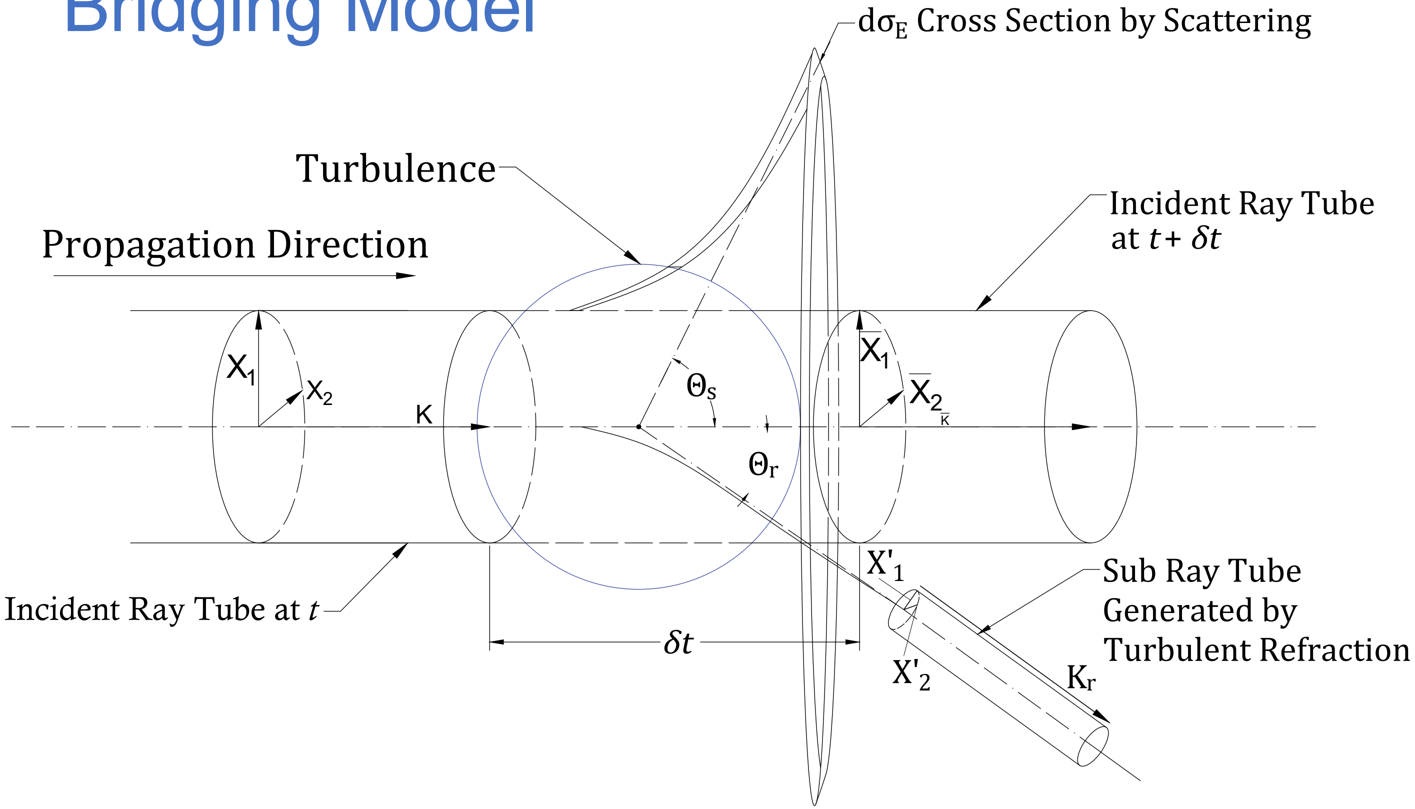
Here we introduce the cross section $\sigma\,(\theta) = \sigma(\boldsymbol{n} - \boldsymbol{n_0}) = \dfrac{\langle I_s \rangle R^2}{I_0 V}$

The final equation for the scattering cross section is

$$\sigma_E\,(\theta) = 2\pi k^4\left[\frac{\beta^2(\theta)\Phi_{\mathrm{T}}(\mathrm{q})}{4T_0^2} + \frac{\beta(\theta)\eta(\theta)\Phi_{\mathrm{CT}}(\mathrm{q})}{2T_0} + \frac{\eta^2(\theta)\Phi_{\mathrm{C}}(\mathrm{q})}{4} + \frac{\boxed{\cos^2\theta\cot^2\frac{\theta}{2}E(q)}}{c_0^2}\right]$$

[1] Ostashev, V. E., and Wilson, D. K. (2015). Acoustics in moving inhomogeneous media (CRC Press).

Bridging Model



Concept of the bridging model

$$\frac{d\boldsymbol{X}_{\boldsymbol{p_i}}}{dt} = \frac{d\overline{\boldsymbol{X}}_{\boldsymbol{pi}}}{dt} + \frac{d{\boldsymbol{X}_{\boldsymbol{p_i}}}'}{dt} = \bar{c}\,\frac{\partial \boldsymbol{N}}{\partial p_i} + \left(\boldsymbol{X}_{\boldsymbol{p_i}} \cdot \nabla\, \bar{c}\right) N + \left(\boldsymbol{X}_{\boldsymbol{p_i}} \cdot \nabla\right)\left(\overline{v_0} + v_0'\right)$$

The convective volume[1] is

$$\nu = \frac{|X_1 \wedge X_2|}{|K|} = |X_1 \wedge X_2| \cdot 2\pi\lambda$$

Implement into the subray tube

$$\nu' = \frac{|X_1' \wedge X_2'|}{|K|} = |X_1' \wedge X_2'| \cdot 2\pi\lambda$$

$$X_i' = \frac{dX'_{p_i}}{dt} = \left(X_{p_i} \cdot \nabla\right) v_0'$$

$$X'_{1,j} = \frac{1}{2} X_{1,i} \left(\frac{\partial\, v_{0,i}'}{\partial\, p_j} + \frac{\partial\, v_{0,j}'}{\partial\, p_i}\right)$$

$$X'_{2,j} = \frac{1}{2} X_{2,i} \left(\frac{\partial\, v_{0,i}'}{\partial\, p_j} + \frac{\partial\, v_{0,j}'}{\partial\, p_i}\right)$$

The new fluctuating ray tube

$$\nu' = \frac{\lambda}{8\pi} X_{1,i} X_{2,i} \overline{\left(\frac{\partial v_{0,i}'}{\partial p_j} + \frac{\partial v_{0,j}'}{\partial p_i}\right)^2} \cos\left(X_1, X_2\right)$$

[1] Gainville, O., "Modeling of Atmospheric Propagation of Infrasound Waves by the Method of Nonlinear Ray Tracing," Ph.D. thesis, Ecole Centrale de Lyon, 2008.

Bridging Model

$$\nu' = \frac{\lambda}{8\pi} X_{1,i} X_{2,i}\, \overline{s_{ij}\, s_{ij}}\, cos(X_1, X_2) \qquad \overline{s_{ij} s_{ij}} = \frac{\epsilon}{2\nu} = \int_0^\infty k^2 E(k)\, dk$$

The turbulent refraction cross-section σ_r is defined as

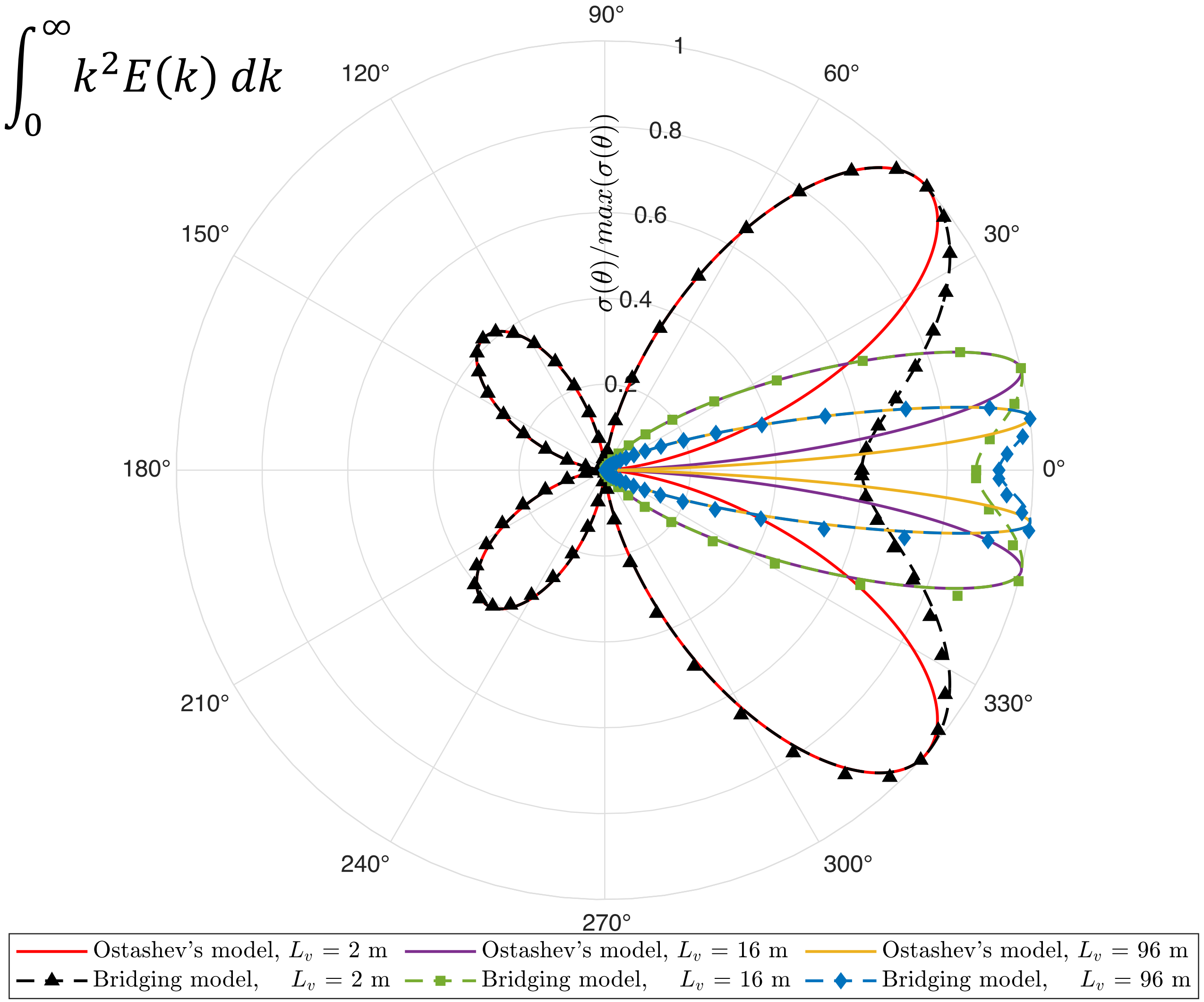
$$\sigma_r = \frac{\nu'}{\nu} \sim \frac{\sigma_v^2}{\lambda_T^2}\left(\int_0^{\frac{2\pi}{L_v}} k^2 E(k)dk \,/ \int_0^\infty k^2 E(k)dk\right)$$

Then a sine function is used to bridge the σ_r and σ_E

$$\sigma_{eff}(\theta) = max(\sigma_E(\theta)) - \frac{A}{\sigma_r}\left[sin\left(\frac{\pi k_r L_v}{2} - \frac{\pi}{2}\right) - 1\right]$$

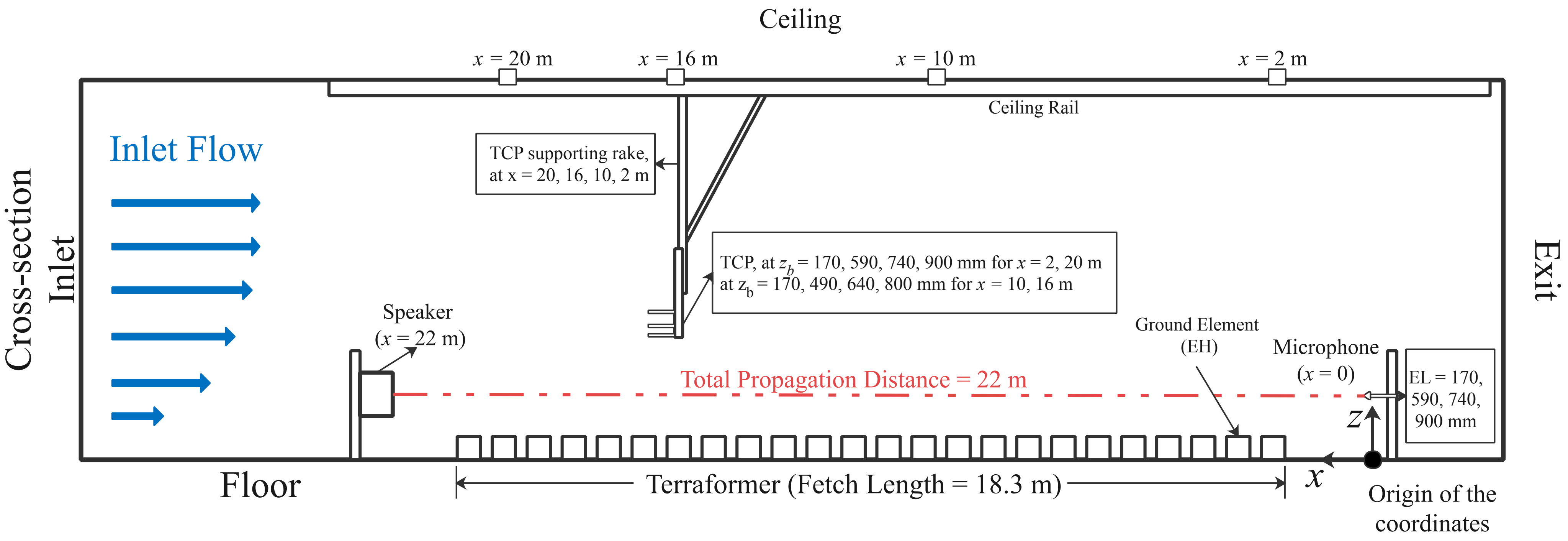
where A is a coefficient to be determined, here we use 1

$$\sigma_{tot} = \sigma_{eff} + \int_0^{2\pi} \int_{2arcsin\left(\frac{\pi}{kL_v}\right)}^{\pi} \sigma_E(\theta) sin(\theta) d\theta\, d\phi$$



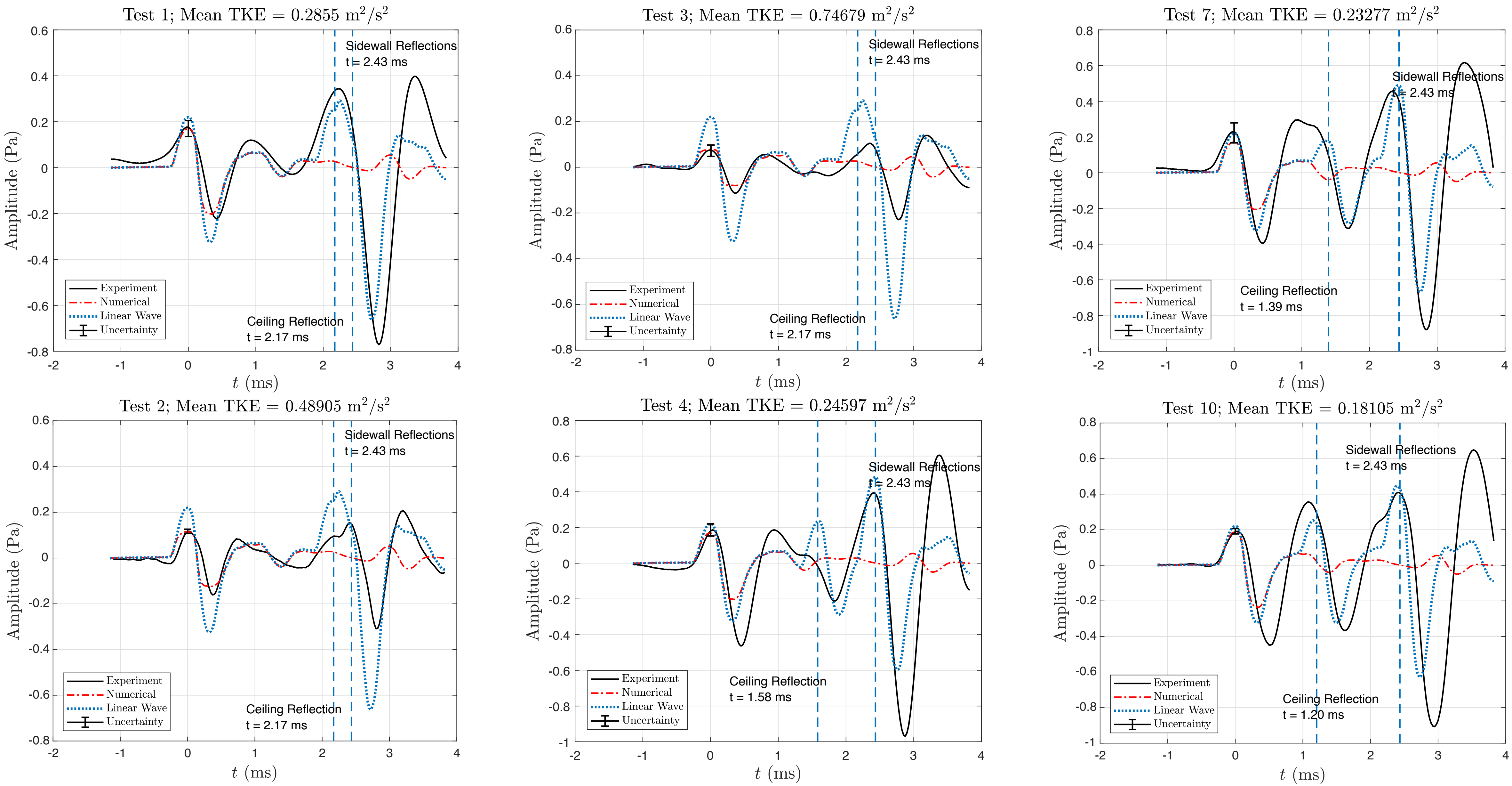
Example output of two models

Experimental Validation



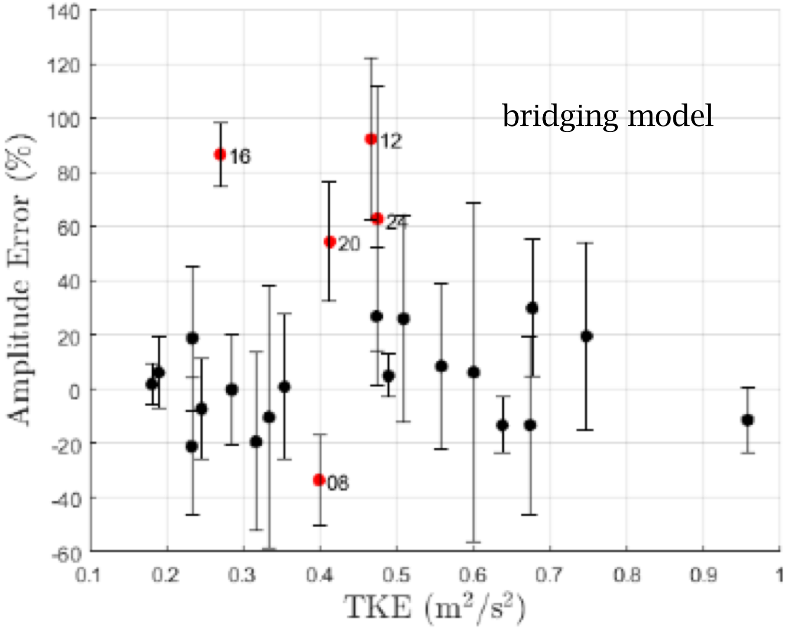
Configuration and experimental procedure of the UFBLWT experiments

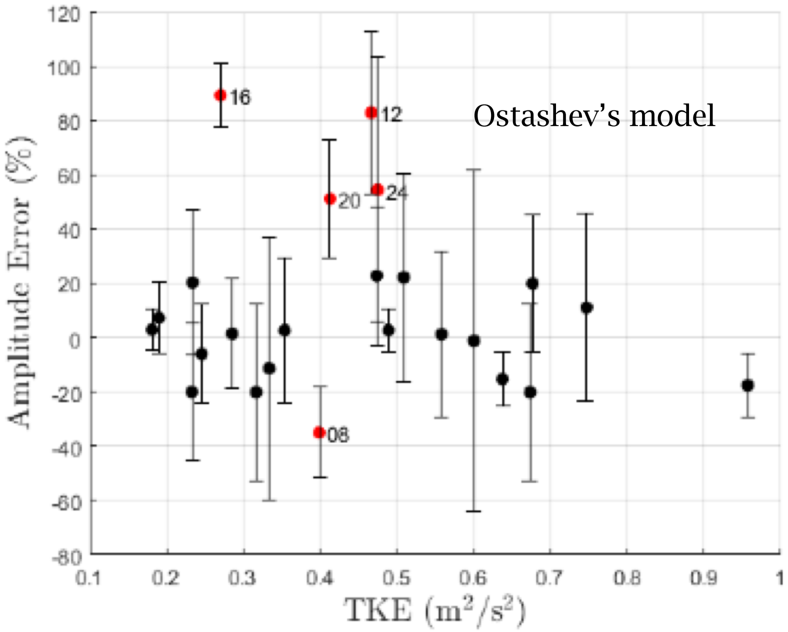
- Reflections are captured by a linear wave propagation solver
- Turbulent attenuation is observed in both experimental data and numerical prediction
- The predictions of the propagation solver is compared with the experimental data



- Reflections are located and isolated while the turbulent attenuation is observed

Experimental Validation





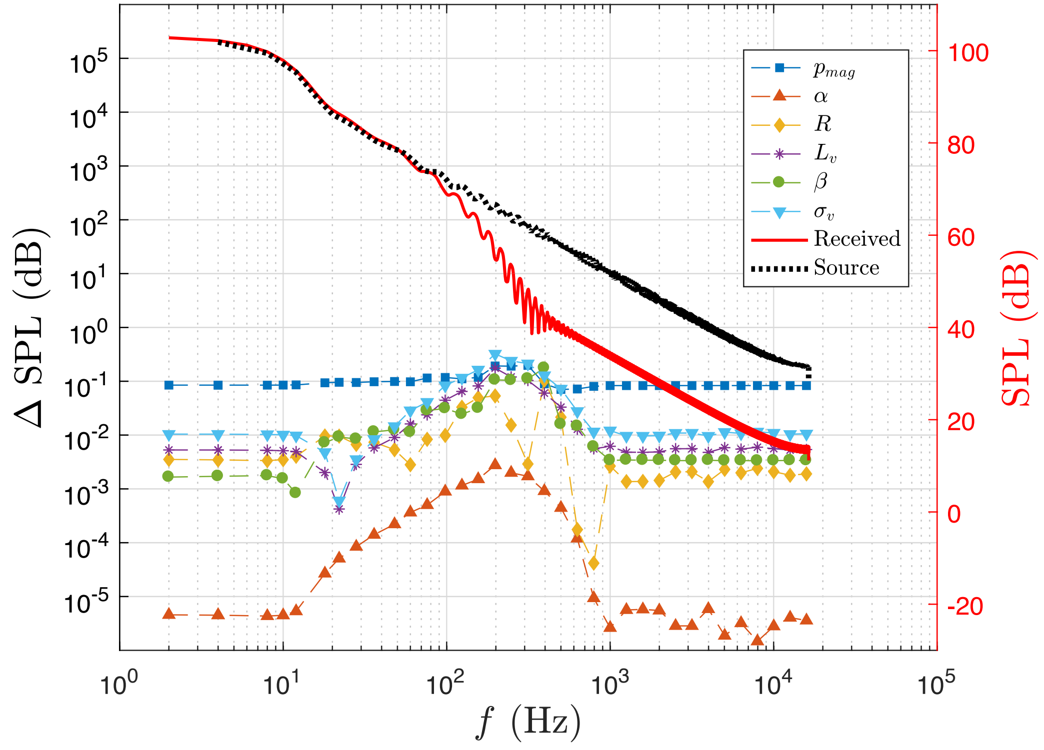
Statistics of the error with the bridging model and Ostashev's model

Table 2. The validation cases

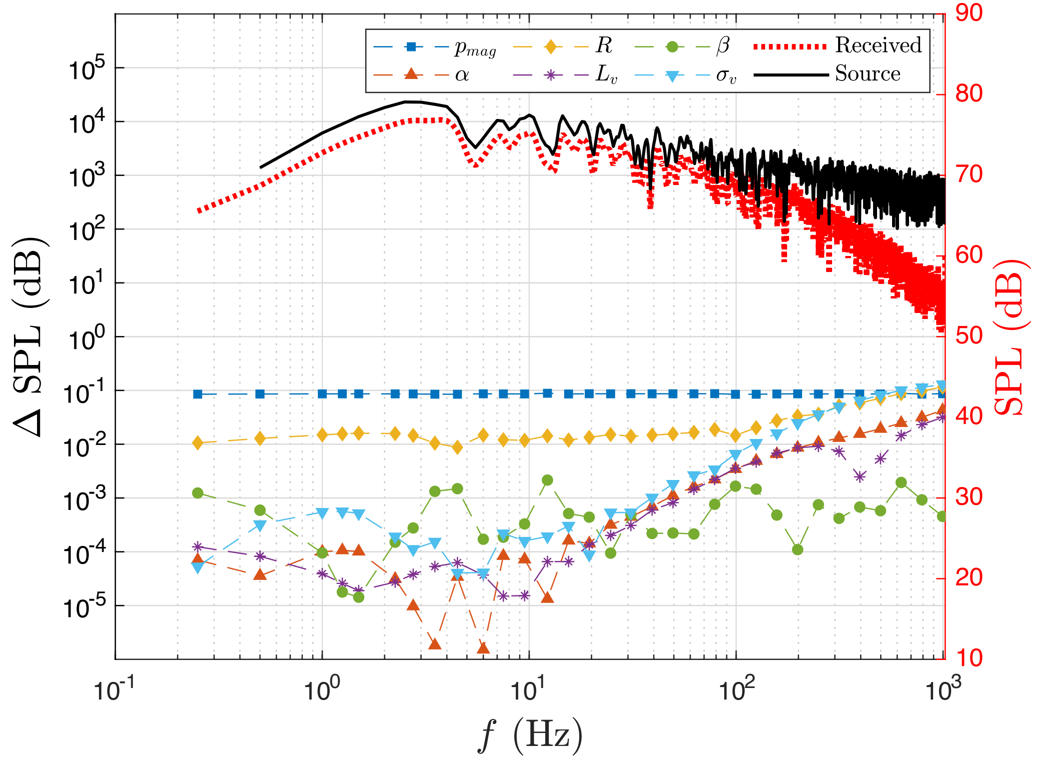
Case #	TKE (m^2/s^2)	L_v (m)	Case #	TKE (m^2/s^2)	L_v (m)
1	0.29	1.89	13	0.35	1.78
2	0.49	2.55	14	0.64	2.40
3	0.75	3.17	15	0.96	2.98
4	0.25	2.00	16	0.27	2.01
5	0.47	2.70	17	0.51	2.70
6	0.67	3.38	18	0.68	3.40
7	0.23	2.04	19	0.23	2.05
8	0.40	2.76	20	0.41	2.78
9	0.56	3.42	21	0.60	3.44
10	0.18	2.06	22	0.19	2.08
11	0.32	2.80	23	0.33	2.81
12	0.47	3.48	24	0.47	3.49

- The overall prediction of the two models agree with each other with error of 22.45% and 24.32%, respectively.
- By removing the outliers, the accuracy of the bridging model is 11.9 %, while the Ostashev's model is 12.67 %.

Sensitivity Analysis



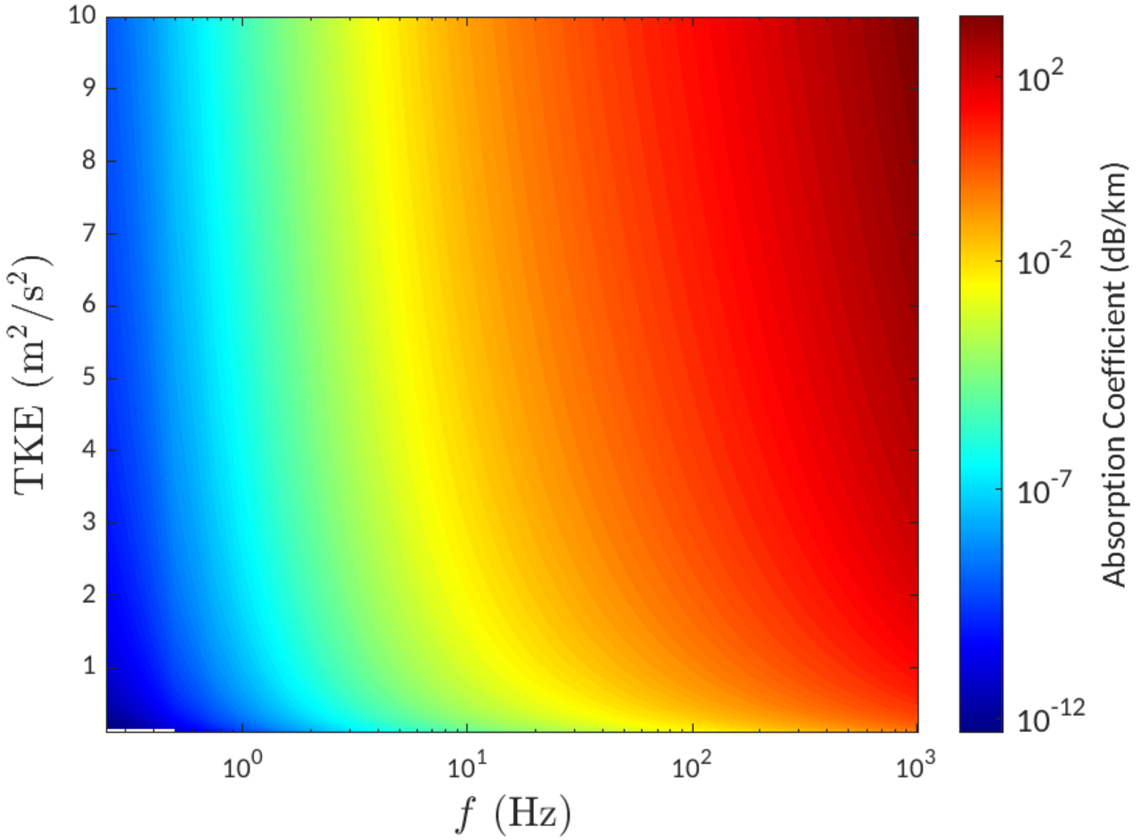
Sensitivity analysis of the shock wave propagation case



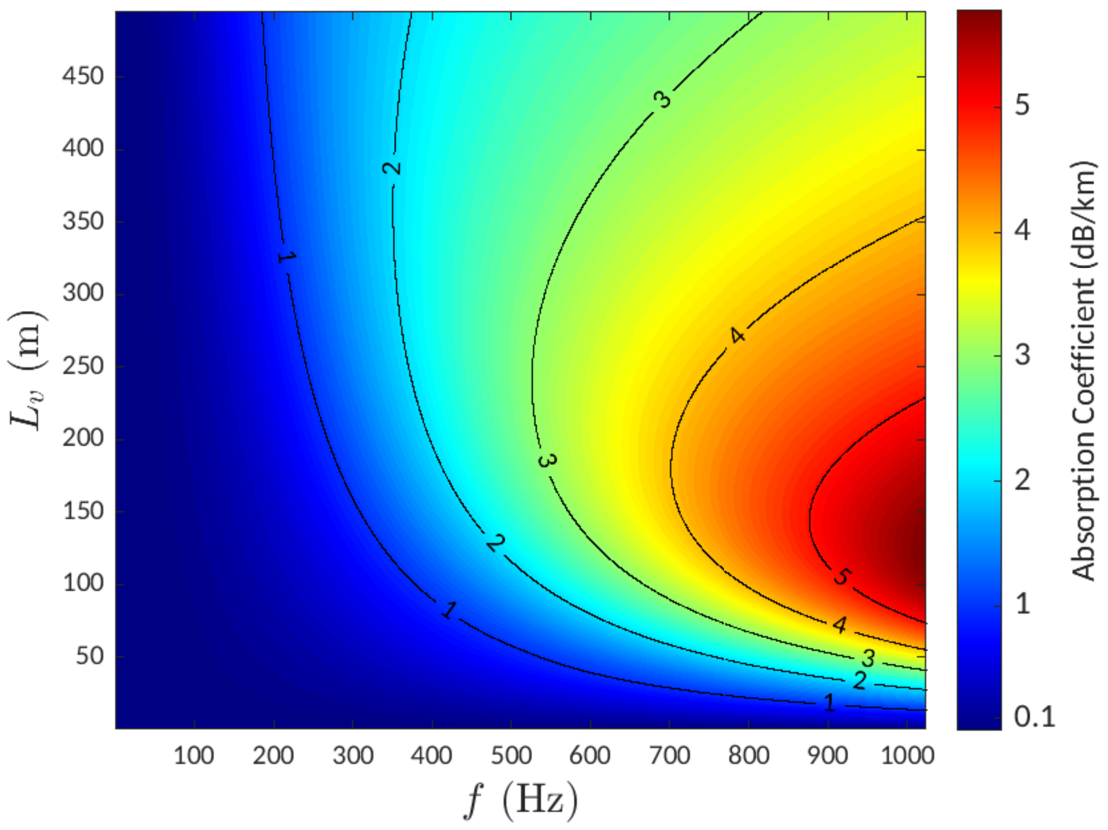
Sensitivity analysis of the tornadic infrasound propagation case

- Turbulent parameters, σ_v and L_v, are compared with other propagation parameters
- In both analysis cases, the turbulent parameters are as sensitive as others

Model Behavior Analysis



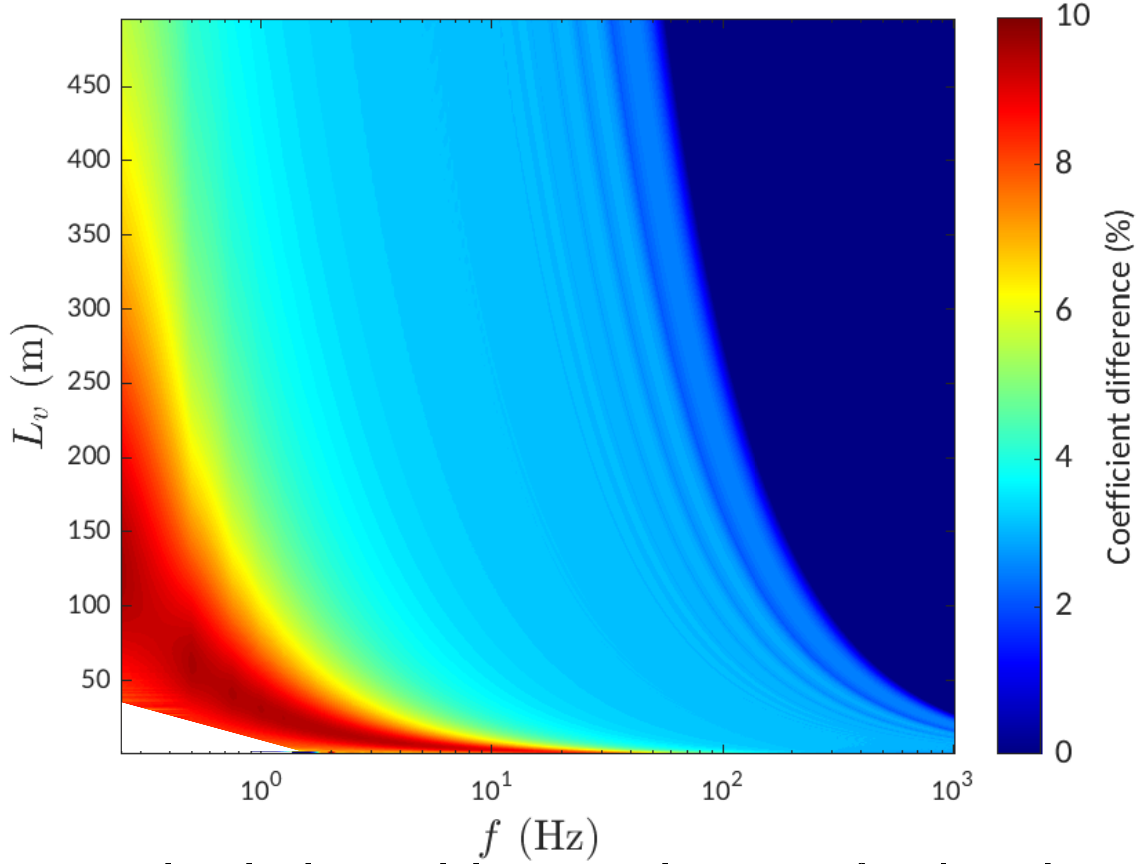
The turbulent attenuation coefficient contour with varying TKE



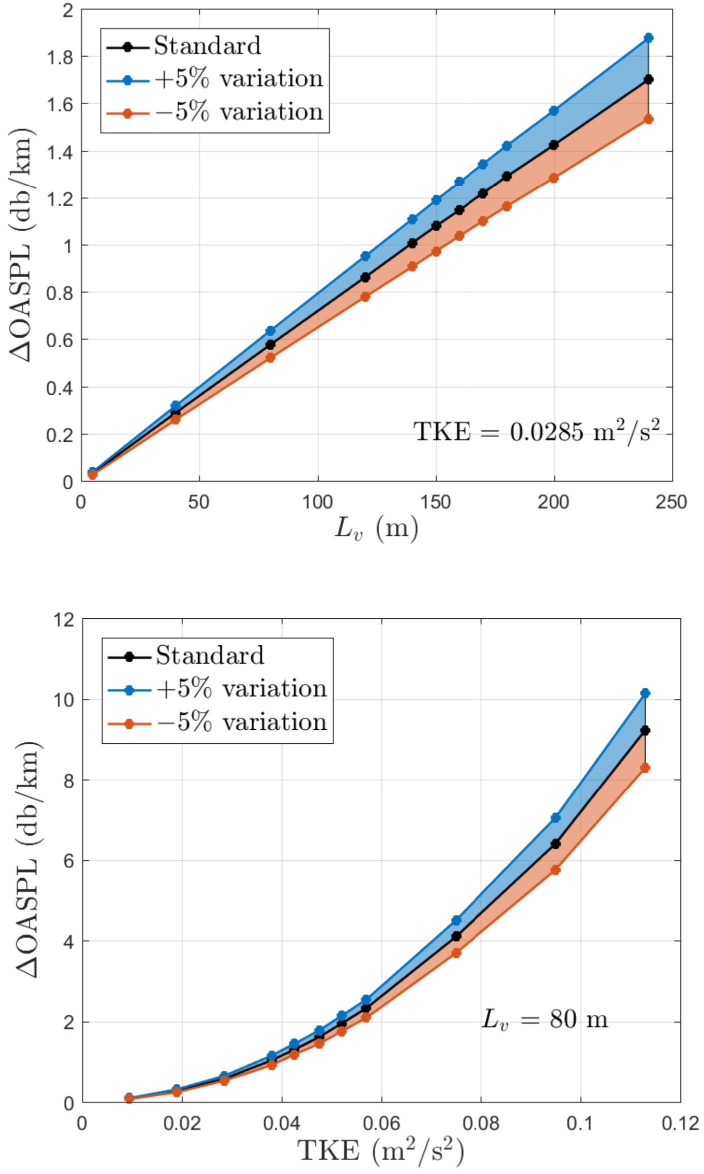
The turbulent attenuation coefficient contour with varying integral length scale

- The turbulent attenuation coefficient increases monotonically with increasing TKE
- There is a maximum turbulent attenuation which is determined by the length scale

Model Behavior Analysis



Anechoic chamber recorded source signal at 2 m away from the speaker



Turbulent effect on OASPL of the tornadic signal

- The difference of the two models is larger in low frequency region
- The TKE and L_v are sensitive on OASPL

Turbulent Attenuation in Realistic Atmosphere

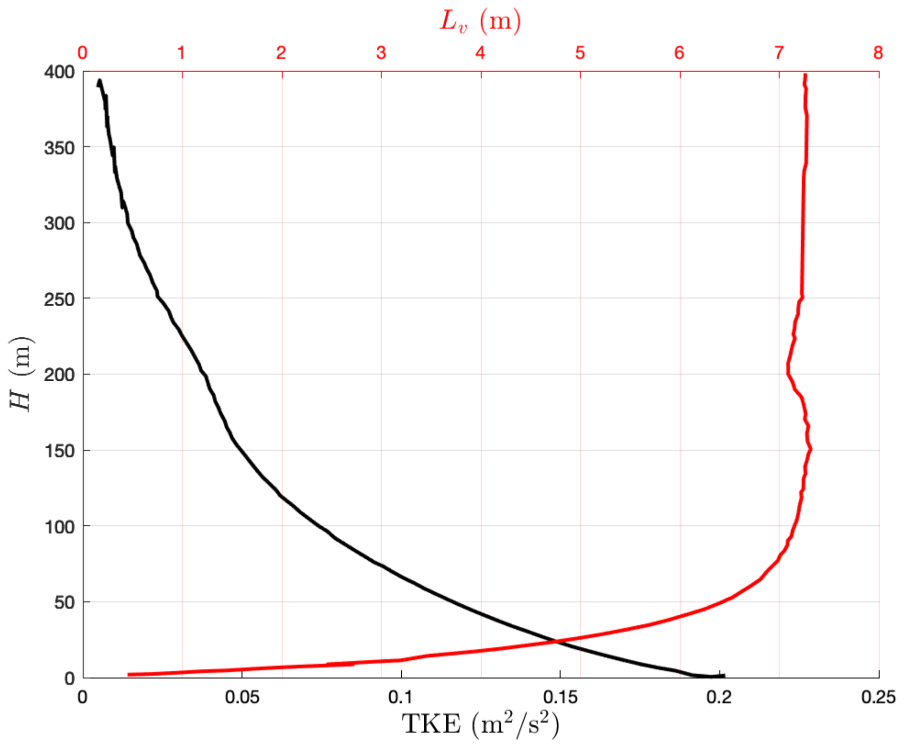


Fig 43. Turbulent kinetic energy and length scale predicted by Apsley's model.

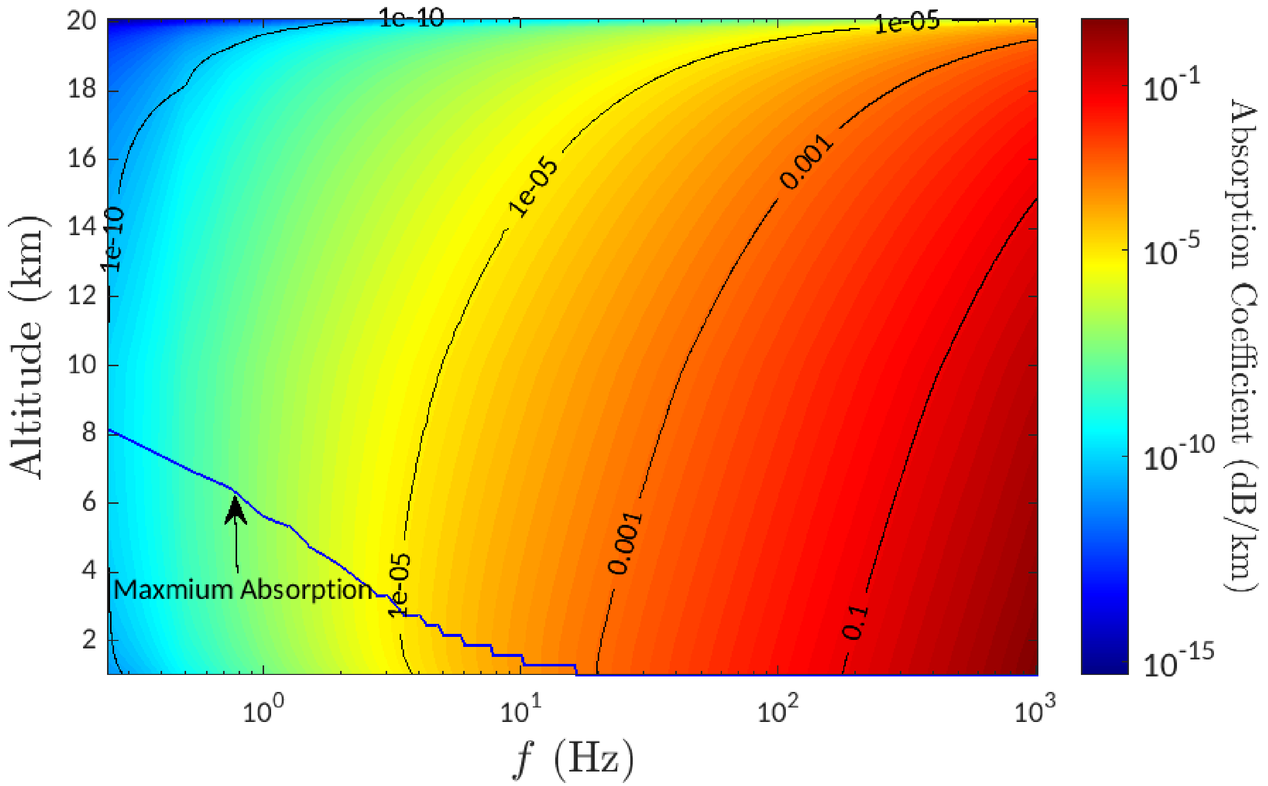
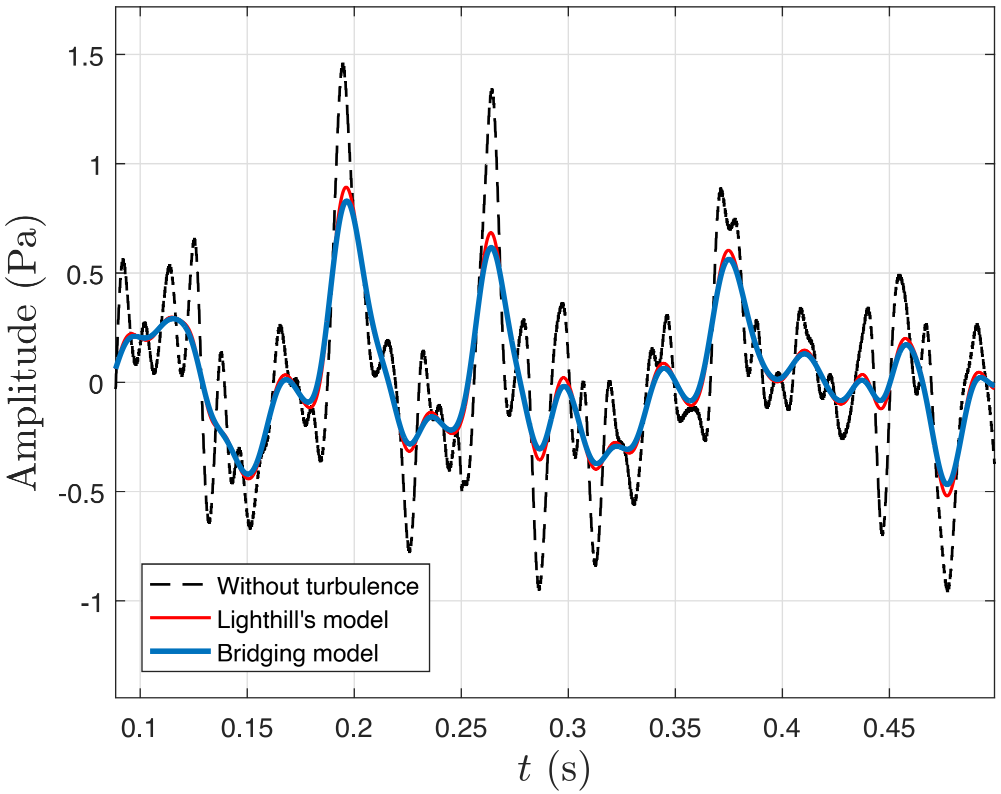


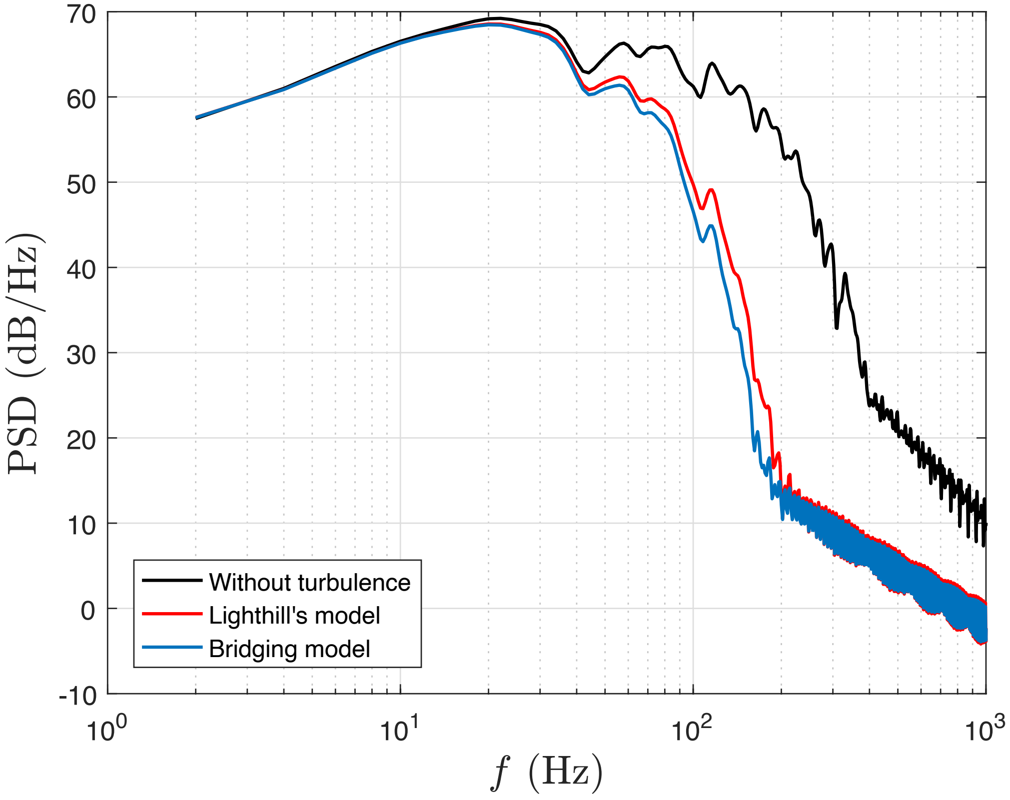
Fig 44. Absorption coefficient distribution in frequency and altitude.

- A realistic atmospheric turbulent model by Apsley is employed
- Turbulent attenuation in model atmosphere is obtained

Tornadic Infrasound Long-Range Propagation



Turbulent kinetic energy and length scale predicted by Apsley's model.



Absorption coefficient distribution in frequency and altitude.

- A realistic atmospheric turbulent model by Apsley is employed

Thank you